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# An Overview of Mobile Robots Applications

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***Abstract*** **- Do we can control of the autonomous robot motion in the more different potential fields? As the answer to this question here it is presented a few methods of the control of the autonomous robots motion in the following fields: a) radial mass density field, b) two-potential electromagnetic and gravitational field, c) control of micro-nano robots motion in multi-potential field, d) quantization of autonomous robot motion in electromagnetic and gravitational fields and e) control of nano-robots motion in multi-potential field. The other possibilities of application of the radial mass density theory to autonomous robot motion are also discussed. The application of the presented theory to control of the micro-nano robot motion is also pointed out. Finally, using the previous methods one can precisely control of the autonomous robot motions on and in the Earth, Ocean and Air.**

***Index Terms*** **— Mobile robots, Multipotential field, Micro - nano robots, Quantization of robot trajectory.**

## I. Introduction

The autonomous robots have a very large application area. The first one is the application in the precise production processes. The second one is in the micro and nano scales as it is in medicine for cell manipulation, drug delivery, medical image acquisition and non-invasive intervention. For that application, one can use the electrical, or chemical actuated robots [1,2-5]. The magnetic soft robots have the advantages because of the fast response, unlimited endurance, and no obstruction restrictions [6]. Here, among the others, the motion of the autonomous robots is described in the radial mass density field. This field is in the region from the minimal radius, with the maximal radial mass density, $\rho_{r\ max}$ , and maximal radius with the minimal radial mass density, $\rho_{r\ min}$. Between these two limited values one can chose $n$ points ($n=1,2,..n_{max}$ ). In the case of the precise robot motion the number $n_{max}$ should be bigger. On the contrary, for the less precise robot motion, the number $n_{max}$ may be smaller. The very important consequence of the solution of the field equations by including gravitational energy-momentum tensor (*EMT*) on the right side of the field equations [7-10] is that the gravitational field exhibit repulsive (positive) and attractive (negative) gravitational forces.

The time transition between quantum states in gravitational field is present in [11]. In order to precisely follow the desired trajectory of the autonomous robot motion one can include the new Relativistic Radial Density Theory (RRDT) [12]. The particle transition and correlation in quantum mechanics is discussed in [13]. Independent position control of two identical magnetic micro-robots in a plane using permanent magnets and magnetically powereful micro robots is presented in [14]. This application represents the new approach to the medical revolution epohe. Magnetically powered micro-robots are discussed in [15,16]. Further, the robust control of micro-robot motion is presented in [17]. The global positioning of robot manipulators with mixed revolute and prismatic joints is discussed in [18]. In the case of vehicle dynamics control, a conjugate gradient-based BPTT-like optimal control algorithm has been applied in [19,20]. The same algorithm can also be adapted to control of the autonomous robot (micro-nano robot) motion in combined electromagnetic and gravitational fields. A robust motion control with antiwindup scheme for electromagnetic actuated microrobot using time-delay estimation is presented in [21].

Further, the quantization of the electromagnetic and gravitational fields is discussed in [22]. The two indipendet position control of two indentical microrobots motion in a plane are realized by using rotating permanent magnets [23]. Magnetically powered microrobots and the robust motion control, with antiwindup scheme for electromagnetic actuated microrobots, are presented in [24] and [25], respectively. Robotics assisted in the minimally invasive surgery process is ilustrated in [26]. Design of a novel haptic joystick for the teleoperation of continuum-mechanism-based medical robots is presented in [27]. In this reference a novel mechanism with series of coupled gears, that aims for the control of continuum robots for medical applications is pointed out. Positioning control of robotic manipulators subject to excitation from non-ideal sources is discused in [28]. Further, a robust motion control with antiwindup scheme for electromagnetic actuated microrobot using time-delay estimation is presented in [29]. Independent position control of two identical magnetic microrobots in a plane using rotating permanent magnets is discused in [30]. Magnetically powered microrobots is pointed out in [31]. A robust motion control with antiwindup scheme for electromagnetic actuated micro-robot , by using time-delay estimation, is presented in [32]. Further, in [33] it is pointed out the robotic assisted minimally invasive surgery. Design of a novel haptic joystick for the teleoperation of the continuum mechanism based medical robots is presented in [34]. The positioning control of robotic manipulators subject to excitation from non-ideal sources is ilustreted in [35]. The so colled tractor-robot cooperation in the heterogeneous leader-follower approach is discussed in [36]. In the reference [37] one can pointed out how the indor positioning systems of mobile robots can be applied. The analysis and experimental evaluation of the single-leg lower-limb rehabilitation robot can applied is point out in [38]. Finaly in the reference [39]

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the multi robot task scheduling for the consensus based fault resilient intelligent behaviour in the smart factories is discussed.

Here it is started with the presentation of the control of dynamics of autonomous robot motion in radial mass density field. In order to solve of the dynamics of the autonomous robot motion the concept of the external linearization of the nonlinear control of the robot motion is applied. It follows by the evaluation of autonomous robot motion in two-potential electromagnetic end gravitational field. The next part of this article gives the method of the quantization of autonomous robot motion in the combination of the electromagnetic and gravitational fields. In development of the control of the autonomous robot motion a concept of the generic state and related orthogonal state are emploed. Finally the control of the nanorobots motion in multipotential field is discussed. In this section it is pointed out that the nanorobotics is the multidisciplinary field with atomic and molecular-sized objects. Therefore sometimes it is called molecular robotics.

## II. Control of Dynamics of Autonomous Robot Motion in Radial Mass Density Field

The problem of the nonlinear control of autonomous robot motion here is discussed as the function of the maximal radial mass density value. In order to simplify the related calculation, here it is started with the concept of the external linearization of the nonlinear control of the robot motion in the radial mass density field. In that case, in the closed regulation loop, one obtains the linear behavior of the hole-system. Thus, the problem of the robot position control in the radial mass density field can be started by the calculation of the control of the error vector, e(t). This vector is a function of the radial mass density, $\rho_r$ , and can be presented by the relations:

$$e = X_w - X,\ \frac{d^2e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c}NF_I\right], \quad (1)$$

$$r_w(t) = \frac{d^2X_w}{dt^2} = \frac{1/n}{\rho_{r\,max} r_{min}}\left[F_{p_w} + F_{t_w} + \frac{1}{c}NF_{Iw}\right].$$

Here $n=1,2,..,n_{max}$ and $n_{max} = \rho_{r\,max} / \rho_{r\,min}$ . Thus in (1) the subscript $w$ denotes the desired robot motion, while the variables without this subscript present the real autonomous robot motion. Further, $Fp$ is a potential force, $Ft$ is a time - variation force, $Fi$ is interaction force and $N$ is the related connection parameter. At the same time the relations (1) also describes the canonical differential equations of autonomous robot motion in the combination of the electromagnetic and gravitational fields. Vector $r_w(t)$ is the desired (nominal) acceleration of the autonomous robot motion.

Now following the idea of the external linearization, one can introduce the next substitution:

$$u(t) = \frac{d^2e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c}NF_I\right], \quad (2)$$

$$u(t) = (u_x(t)\, u_y(t)\, u_z(t))^T.$$

Here $u(t)$ is the internal control vector of autonomous robot motion in radial mass density field. Further, applying the phase state-space variables, $(z_1\ z_2\ z_3)^T$ in (1), we obtain the related state-space model of the robot motion in the radial mass density field:

$$e = (e_x\ e_y\ e_z)^T = Z_I = (z_1\ z_2\ z_3)^T,$$

$$\frac{de}{dt} = (\frac{de_x}{dt}\ \frac{de_y}{dt}\ \frac{de_z}{dt})^T = Z_{II} = (z_4\ z_5\ z_6)^T, \quad (3)$$

and

$$dZ/dt = AZ(t) + Bu(t),\ A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ I \end{bmatrix}, I = diag[1,\ 1,\ 1]. \quad (4)$$

In (4), parameters $A$ and $B$ are constant matrices with dimension (6x6) and (6x3), respectively. Here, it is supposed that the disturbances in state-space model of the robot motion in the radial mass density field (3 ) and (4) are of the initial condition types. In order to eliminate the control error of the autonomous robot motion in the radial mass density field (caused by the disturbances) one can introduce the following internal control:

$$F_p = \frac{1}{n}\rho_{r\,max} r_{min}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right]$$

$$-\left[F_t + \frac{N}{c}F_I\right],\ u(t) = -KZ. \quad (5)$$

Here, $K$ is the state space controller, $Z$ is control error, $Fp$ is the potential force, $Ft$ is time variable force, $F_I$ is an interaction force, $N$ is a constant and $c$ is the speed of the light in vacuum. Including the internal control relations (3) and (4) into (5), one obtains the related equation of the potential force as function of radial mass density value in the linear form:

$$F_p = \frac{1}{n}\rho_{r\,max} r_{min}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left[F_t + \frac{N}{c}F_I\right]. \quad (6)$$

Now starting from the previous relations one can generate the new equations of the potential forces $Fp$ :

$$F_{p_x} = -\left(\frac{\partial \Sigma U_j}{\partial x} + \frac{\partial U_c}{\partial x}\right),\ F_{p_y} = -\left(\frac{\partial \Sigma U_j}{\partial y} + \frac{\partial U_c}{\partial y}\right),$$

$$F_{p_z} = -\left(\frac{\partial \Sigma U_j}{\partial z} + \frac{\partial U_c}{\partial z}\right). \quad (7)$$

It is follows by the inclusion of the control potential force, $Fcp$ , that is derived from the artificial control field with potential control energy $Uc$ . After inclusion of (7) into (6), one obtains the nonlinear control of the autonomous robot (micro – nano robot) motion in the multi-potential field as the function of the maximal radial mass density value, denoted by $\rho_{r\,max}$ :

$$F_{cp} = \frac{1}{n}\rho_{r\,max} r_{min}\left[r(t) + K_I Z_I + K_{II} Z_{II}\right]$$

$$-\left[F_{dp} + F_t + \frac{N}{c}F_I\right]. \quad (8)$$

Now, using (8), the control of the nonlinear system is solved in the radial mass density field by employing the concept of the external linearization as the function of the maximal radial mass density value $\rho_{r\,max}$ .

## III. Dynamics of Autonomous Robot Motion in Two Potential Electromagnetic and Gravitational Field

The general approach to control of the dynamics of the autonomous robot motion in radial mass density fields (8), can also be applied to the two-potential electromagnetic and gravitational field. In this sense, let an autonomous robot be an electricaly charged particle with charge $q$ and rest mass $m_0$ that is moving with a non-relativistic velocity (v << c). Here it is also assumed that the gravitational field is produced by the spherically symmetric non charged body with mass $M$ and total potential energy $U$. For calculation of the autonomous robot motion in the two potential radial mass density fields one can use the following relation:

$$U = qV_e + m_0 V_g = qV_e + m_0\left(-\frac{GM}{r}\right),$$
$$\rho_{r\,max} = \frac{m_0}{r_{min}},\ U = qV_e + \frac{1}{n}\rho_{r\,max} r_{min}\left(-\frac{GM}{r}\right). \quad (9)$$

Here $Ve$ and $Vg$ are the related scalar potentials of the electromagnetic and gravitational radial mass density fields, respectively. Parameter $G$ is the gravitational constant and $r$ is the radius as distance between the autonomous robot and center of the mass $M$. Here $n=1,2,..,n_{max}$ , $n_{max} = \rho_{r\,max} / \rho_{r\,min}$ . Now applying (9) and using the notations, ($Ee,He$) for an electromagnetic field and ($Eg,Hg$) for the gravitational field, one can generate the vector equation as the explicit functions of the Lorentz forces:

$$\frac{1}{n}\rho_{r\,max} r_{min}\frac{d^2X}{dt^2} = q\left(E_e + \frac{1}{c}v\times H_e\right)$$
$$+\frac{1}{n}\rho_{r\,max} r_{min}\left(E_g + \frac{1}{c}v\times H_g\right). \quad (10)$$

The parameters $Ee$, $Eg$, $He$ and $Hg$ are vectors described by the relations:

$$E_e = \begin{bmatrix} E_{e_x} \\ E_{e_y} \\ E_{e_z}\end{bmatrix},\ E_g = \begin{bmatrix} E_{g_x} \\ E_{g_y} \\ E_{g_z}\end{bmatrix},\ H_e = \begin{bmatrix} H_{e_x} \\ H_{e_y} \\ H_{e_z}\end{bmatrix},\ H_g = \begin{bmatrix} H_{g_x} \\ H_{g_y} \\ H_{g_z}\end{bmatrix}. \quad (11)$$

In this example, an autonomous robot is a particle with charge $q$ and rest mass $m_0$. Further, let the autonomous robot interacts with both electromagnetic and gravitational radial mass density fields. In that sence the relations (10) and (11) describe the dynamic of the autonomous robot motion in two-potential electromagnetic and gravitational field. The components of the vector $Ee$ and $Eg$ can be calculated by using the following equations:

$$E_{e_x} = -\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t},\ E_{g_x} = -\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t},$$
$$E_{e_y} = -\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t}, \quad (12)$$

and:

$$E_{g_y} = -\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t},\ E_{e_z} = -\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t},$$
$$E_{g_z} = -\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}. \quad (13)$$

The components of vectors $Ae$, $Ag$, $He$ and $Hg$ in (12) and (13) are given by the following relations:

$$A_{e_i} = \left(\frac{v_i V_e}{c}\right),\ A_{g_i} = \left(\frac{v_i V_g}{c}\right),\ i = x,y,z,$$
$$H_{e_x} = \frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z},\ H_{g_x} = \frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}, \quad (14)$$

and:

$$H_{e_y} = \frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x},\ H_{g_y} = \frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x},$$
$$H_{e_z} = \frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y},\ H_{g_z} = \frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}. \quad (15)$$

Applying (14) and (15) to the canonical differential equations of the autonomous robot motion in the two-potential radial mass density field, and using $m_0 = \rho_{r\,max} r_{min}$ one obtains the control error model of the autonomous robot motion as a function of the maximal radial mass density value:

$$d^2e/dt^2 = r_w(t) - \frac{q}{\frac{1}{n}\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}\mathrm{v}\times H_e\right)$$
$$-\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right), \quad (16)$$

and:

$$r_w(t) = \frac{q}{\frac{1}{n}\rho_{r\,max} r_{min}}\left(E_{e_w} + \frac{1}{c}\mathrm{v}_w\times H_{e_w}\right)$$
$$-\left(E_{g_w} + \frac{1}{c}\mathrm{v}_w\times H_{g_w}\right). \quad (17)$$

In (16) and (17) $r_w(t)$ is the vector of the desired acceleration of the autonomous robot motion. The subscript $w$ denotes desired values of the related variables. The next step is the application of the concept of the external linearization in order to transform (16) into the new relation:

$$u(t) = r_w(t) - \frac{q}{\frac{1}{n}\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}v\times H_e\right)$$
$$-\left(E_g + \frac{1}{c}v\times H_g\right). \quad (18)$$

Here $u(t)$ is the internal control vector and $n=1,2,..n_{max}$ is the number of the robot steps from the minimal to the maximal radiuses in radial mass density field. From (17) and (18), one obtains the related equivalent of the linear control error model of the autonomous robot motion in the combined electromagnetic and gravitational radial mass density field given by (14) and (15), respectively. Now the phase state-space variables of the system (2) are determined by applying the relation (3). The presented state-space model of an autonomous robot motion is given in the matrix form (6).

In order to eliminate the control error of an autonomous robot motion, caused by disturbances of the initial condition types, one can introduce internal control in the form (7). Applying (7) to (18), one obtains the new relation as the function of the maximal radial mass density value in the form:

$$E_e = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(\frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (19)$$

Now, let the electric field *Ee* is consisting of the two electric components *Ee* = *Ede* + *Ece*. Here *Ede* is a disturbance electric field that is caused by the influence of the two-potential field to the motion of the autonomous robot in radial mass density field. The component *Ece* is an artificial electric control field that should control autonomous robot motion in the two potential field. Including *Ee* from (19), one obtains the nonlinear electric control of the autonomous robot motion in the two-potential radial mass density field, as the function of the maximal radial mass density $\rho_{r\,max}$:

$$E_{ce} = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(E_{de} + \frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (20)$$

Taking into account the relation (10), the canonical differential equations of the autonomous robot motion, in the two-potential radial mass density field, can be rewritten as a function of the maximal radial mass density value:

$$\frac{d^2X}{dt^2} = \frac{nq}{\rho_{r\,max} r_{min}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (21)$$

Applying the nonlinear control *Ece* from (20) to the nonlinear dynamical model of the autonomous robot motion (21), one obtains the closed-loop system in the linear form:

$$\frac{d^2X}{dt^2} = r_w(t) + K_I Z_I + K_{II} Z_{II}\,. \quad (22)$$

Thus, the equation (20) is the relation for the nonlinear control, which in the closed loop with a nonlinear canonical differential equations of autonomous robot motion (21), results in the linear behavior of the hole system (22). On that way the problem of control of the autonomous robot motion in the combination of an electromagnetic and gravitational radial mass density field, has been solved by employing the so-called concept of the external linearization. This is very important for application to the micro and nano robots in the applications to the drag delivery across the human body. Of cose the robots in the combination of the electromagnetic and gravitational radial mass density field can also be applied to the large areas of production engineering systems.

## IV. Quantization of Autonomous Robot Motion in Electromagnetic and Gravitational Fields

In order to quantize the mobile robot (micro-nano robot) motion in the combination of electromagnetic and gravitational fields, one can start with the related equations [22]:

$$\frac{M_g}{L_{g\min}} = \frac{M_p}{L_p} = \frac{c^2}{G} = const.\;\; L_{g\min} = \frac{GM_g}{c^2} = 2r_{\min},$$
$$L_{g\min} = \frac{2GM_p}{(1+\kappa)c^2} = \frac{2m_0\,GM_g}{\left(\pm qV_e - U_{l_{\min}}\right)} = const., \quad (23)$$
$$U_{l_{\min}} = \pm qV_e - 2m_0 = const.$$

Here $r_{min}$ is a minimal radius, $L_{gmin}$ is a minimal length, *c* is the speed of the light in vacuum, *Mp* is Planck's mass, *Lp* is Planck's length, *Mg* is a gravitational mass, *Ve* is the electrical potential, *q* is the electrical particle charge, *G* is a gravitational constant and $U_{Lmin}$ is a potential energy at the minimal length. The total energy is maximal at the minimal length and is limited by the electrical potential energy $qV_e$. Using Planck's mass and Planck's length it is calculated the new parameter *κ*. This parameter is the energy conservation constant with value κ = 0.99993392118.

The quantization of the combined electromagnetic and gravitational fields can be realized by using the procedure given in [22]:

$$\frac{2L_{g\min} - L_{g\min}}{L_d} = \frac{2m_0\,GM_g}{L_d\left(\pm qV_e - U_{l_{\min}}\right)} = \frac{2GM_p}{L_d(1+\kappa)c^2} = n.$$
$$n = 1,2,..,n_{\max},\;\; n_{\max} \le \frac{4GM_g}{\pi\hbar c^3}\,\frac{\Delta H^\wedge}{(1+\kappa)}. \quad (24)$$

Here $L_d$ is the minimal distance between two quantum points, $H^\wedge$ is the energy uncertainty and $\Delta T$ is the shortest time during which the average value of a certain physical quantity is changed by an amount equal to the standard deviation or uncertainty of time. This time should satisfy the condition, given in [13]:

$$\Delta H^\wedge \Delta T \ge \frac{\hbar}{2},\; \Delta_\psi H^\wedge = \left(\langle\psi|H^{\wedge 2}|\psi\rangle - \langle\psi|H^\wedge|\psi\rangle^2\right)^{1/2},$$
$$\eta_t \equiv t_{\min}/\tau_{CQS},\;\; \eta_{\psi\to\psi\perp} \equiv \frac{\tau_{\psi\to\psi\perp}}{\tau_{CQS}},$$
$$\eta_{\psi\to\psi\perp} = \frac{\pi\hbar}{2\Delta H^\wedge \tau_{CQS}},\;\; \tau_{\psi\to\psi\perp} \ge \pi\hbar/2\Delta H^\wedge. \quad (25)$$

The quantum dynamical evolutions (24,25) are starting from a generic state $|\psi\rangle$ and is finishing in the related orthogonal state. The quantitative measure of temporal quantum state transfer efficiency is denoted by $\eta_t$ and by the shortest physically possible time $t_{\min}$ in order to obtain the quantum transition between two quantum states. Parameter $\tau_{CQS}$ can be stated as the time, effectively spent by the controlled system or control algorithm. Parameter $\tau_{\psi\to\psi\perp}$ is the shortest physically possible time that is spent for the transition to the state $\psi_{\perp}$. This is the minimum transition time between two quantum points. For determination of the minimal time in quantum dynamical evolution, one should use time depended Hamiltonian. In that case, the time-energy uncertainty relation can be used. The application of the previous theory to the mobile robot (micro-nano robot) quantum motion can be started with the minimal distance $L_{dmin}$ between two quantum states (24, 25). Let the maximal transition velocity between two quantum points is less than the speed of the light in a vacuum. In that case, the minimal distance $L_{dmin}$ and the maximal number of quantum points $n_{max}$ in the region $2L_{min}$-$L_{min}$ are determined by the relations:

$$v < c \rightarrow L_{d\min} = v\,\tau_{\psi\rightarrow\psi\perp},\; n_{\max} < \frac{2L_{g\min} - L_{g\min}}{v\,\tau_{\psi\rightarrow\psi\perp}},$$

$$n_{\max} \le \frac{m_0\, GM_g}{\left(\pm qV_e - U_{l_{\min}}\right)} = \frac{2GM_p}{L_{d\min}(1+\kappa)v^2}. \quad (26)$$

For the precise motion of the autonomous robot (micro/nano robot) one can introduce desired velocity v and minimal distance Ldmin . In that case, from (26) one obtains the value of the maximal number of quantum points nmax in the region 2Lmin-Lmin :

$$v = 10^{-6}\frac{m}{s},\quad L_{d\min} = \frac{10^{-6}}{2}\,m,$$

$$n_{\max} \le \frac{2GM_p}{(L_{d\min}/2)(1+\kappa)v^2} = 2.90533. \quad (27)$$

From (27) one can see that the minimal distance between two quantum states (step of autonomous robot or micro-nano robot moving) is equal to $0.5\cdot10^{-6}$ m = 0.5 microns and the maximal number of quantum points $n_{max}$ is less or equal to 2.90533.

## V. Control of Nanorobots Motion in Multipotential Field

The nanorobotics is the multidisciplinary field with atomic and molecular-sized objects and therefore sometimes is called molecular robotics [23]. The state of the art in nanorobotics has been presented in [24]. The one of approaches for building useful devices from nanoscale components is presented in [25]. Generally, nanorobots have various mechanical components such as nanogears built primarily with carbon atoms in a diamondoid structure. The second line of nanorobotics research involves manipulation of nanoscale objects with macroscopic instruments and related potential fields. The all of these instruments are collectively known as Scanning Probe Microscopes (SPMs). For more information on SPM technology one can see the references [26]. The spatial region in nanorobotics is the bionanorobotics [27]. The main goal in this region is to develop novel and revolutionary biomolecular machine components that can be assembled and form multi-degree of freedom nanodevices [27]. These bionanodevices should be able to apply forces and manipulate objects in the nanoworld, transfer information from the nano to the macro world, receive the information from the macro world and also be able to travel in the nano environment. In order to control nanorobots in mechanics, electronics, electromagnetic, photonics, chemical and biomaterials regions we have to have the ability to construct the related artificial control potential fields. Thus, the first step in designing the control dynamics for nanorobots is the development of the relativistic Hamiltonian that will include external artificial potential field. This Hamiltonian has been derived and presented in [28].

Let the non-relativistic approximation of the Hamiltonian $H$ for a nanorobot motion in a multipotential field is given by the relation:

$$H \cong m_0c^2 + \frac{1}{2m_0}\left[\left(p_x - \frac{v_x U}{c^2}\right)^2 + \left(p_y - \frac{v_y U}{c^2}\right)^2 + \left(p_z - \frac{v_z U}{c^2}\right)^2\right] + U. \quad (28)$$

Here $m_0$ is a rest mass of a nanorobot, $c$ is a speed of the light in a vacuum, $p_x$, $p_y$, and $p_z$, as well as $v_x$, $v_y$ and $v_z$ are momentums and velocities, respectively, in $x$, y, and $z$ directions and $U$ is a total potential energy of a nanorobot in a multipotential field. In the relation (28) $U$ is a potential energy of the nanorobot in the $j$-th potential field. In the case where there are no quantum mechanical effects one can employ classic Hamiltonian canonic forms for designing equations of the nanorobot motion:

$$\dot{p}_x = -\frac{\partial H}{\partial x},\quad \dot{p}_y = -\frac{\partial H}{\partial y},\quad \dot{p}_z = -\frac{\partial H}{\partial z},\quad \dot{x} = \frac{\partial H}{\partial p_x},$$
$$\dot{y} = \frac{\partial H}{\partial p_y},\quad \dot{z} = \frac{\partial H}{\partial p_z}. \quad (29)$$

Now, one can define the so called interaction terms of a nanorobot motion in a multipotential field:

$$I_x = v_xU/c,\quad I_x = v_yU/c,\quad I_z = v_zU/c \quad (30)$$

It follows the definition of the interaction forces as functions of the interaction terms:

$$F_{I_x} = \frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z},\quad F_{I_y} = \frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x},\quad F_{I_z} = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}. \quad (31)$$

The next definition is related to the time-varying forces as the functions of the interaction terms:

$$F_{t_x} = -\frac{1}{c}\frac{\partial I_x}{\partial t},\quad F_{t_y} = -\frac{1}{c}\frac{\partial I_y}{\partial t},\quad F_{t_z} = -\frac{1}{c}\frac{\partial I_z}{\partial t}. \quad (32)$$

Finally, one can define the potential forces as the function of the total potential energy of a nanorobot in a multipotential field:

$$F_{p_x} = -\frac{\partial U}{\partial x},\quad F_{p_y} = -\frac{\partial U}{\partial y},\quad F_{p_z} = -\frac{\partial U}{\partial z}. \quad (33)$$

Applying (1) to (4) and including the relations (5), (6), (7) and (8), one obtains the compact form of the canonical differential equations of the nanorobot motion in a multipotential field as the functions of the mentioned forces:

$$
\begin{aligned}
m_0\ddot{x} &= F_{p_x} + F_{t_x} + \frac{1}{c}\left(\dot{y}F_{I_z} - \dot{z}F_{I_y}\right),\\
m_0\ddot{y} &= F_{p_y} + F_{t_y} + \frac{1}{c}\left(\dot{z}F_{I_x} - \dot{x}F_{I_z}\right),\\
m_0\ddot{z} &= F_{p_z} + F_{t_z} + \frac{1}{c}\left(\dot{x}F_{I_y} - \dot{y}F_{I_x}\right).
\end{aligned} \qquad (34)
$$

Following the previous consideration one can introduces the following vectors:

$$
\begin{aligned}
&X = \left[x\ \ y\ \ z\right]^T,\ \dot{X} = \left[\dot{x}\ \ \dot{y}\ \ \dot{z}\right]^T,\ \ddot{X} = \left[\ddot{x}\ \ \ddot{y}\ \ \ddot{z}\right]^T,\\
&F_p = \left[F_{p_x}\ F_{p_y}\ F_{p_z}\right]^T,\ F_I = \left[F_{I_x}\ F_{I_y}\ F_{I_z}\right]^T,\ F_t = \left[F_{t_x}\ F_{t_y}\ F_{t_z}\right]^T.
\end{aligned} \qquad (35)
$$

Including the vectors (35) into the relations (34) one generates the vector-matrix form of the canonical differential equations of the nanorobot motion in a multipotential field:

$$
m_0\ddot{X} = F_p + F_t + \frac{1}{c}NF_I,\quad N = \begin{bmatrix} 0 & -\dot{z} & \dot{y} \\ \dot{z} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}. \qquad (36)
$$

Thus the relation (36) can be applied to the control of the nanorobots motion in the multipotential field.

## VI. Conclusion

The presented theory is applied to the control of the robot motion in the multy potential fields. In that sense it is started with the control of the autonomous robot motion in the radial motion from the minimal to the maximal gravitational radiuses and vice-verse. It is shown that the Planck's and gravitational parameters can be described as the functions of the radial mass density values. In that case the maximal radial mass density occurs at the minimal gravitational radius of the related mass. On the other hand, the minimal radial mass density is happened at the maximal radius of the related mass. Further, the quantization of the autonomous robot (micro-nano robot) motion in the combination of electromagnetic and gravitational fields is presented. In the case of the precise motion control of the autonomous robots in the gravitational radial direction one can use the variable step of the robot motion. The presented methods of the control of the autonomous robot motion can be applied on and in the Earth, Oceans and Air.

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