International Journal of New Technology and Research (IJNTR)
ISSN: 2454-4116, Volume-8, Issue-8, August 2022 Pages 06-08

Deriving the Sum of a Harmonic Progression

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***Abstract*— In mathematics, mathematicians and statistics have always been fascinated by the nature of numbers. Since the birth of mathematics, a lot has been there that can be done by the use of numbers. There is still more to do with numbers. This involves finding patterns that can be used to relate numbers. This makes the numbers more interesting to play with and discover more fascinating formulas and ideas. In this article, a lot will be discussed on number patterns, the relationship between several number progressions, and the formulas associated with them. More specifically, the sum of harmonic progression and its applications. To be specific, harmonic progression is a number sequence generated by obtaining the reciprocals of the real numbers in an arithmetic progression.**

***Index Terms*— Harmonic Progression, statistics.**

I. Introduction

A number pattern is a trend followed by a list of numbers in a given sequence. Patterns are what define the relationship between two numbers. Solving a sequence requires an understanding of the rule applied in getting the next term or the sum of given terms. The understanding is required in solving all the types of number series in the field of mathematics. According to (Andrews, 1990), a number pattern is a generalization of the rule followed by a series of numbers.

Before we understand harmonic progression, a scholar will always want to know what progression is. A progression is a pattern of numbers. For instance, $2, 4, 8, \ldots$ gives a progression since there is an observed pattern that the numbers follow. In this progression, the next number is gotten by adding 2 to the previous number. However, a pattern depends on the type of progression.

As indicated, a progression is a real number list exhibiting a particular trend. Every term in a progression is given by a generalized rule called the nth term denoted by a_n.

For our example of a progression, $2,4,6,8,\ldots$the nth term can be given by the generalized formula $2n+2$. When $n=0$, then the first term is 2, when $n=1$, then the second term becomes 4, and so on, this is a perfect generalized rue for obtaining that next or nth term in this progression. With or focus on harmonic progression, we have several other types of progressions that includes; arithmetic progression, and geometric progression (Panagiotou, 2011). Because the harmonic sequence is the reciprocal of the arithmetic progression, we need to understand the arithmetic progression to define the harmonic progression perfectly. A given sequence is said to be a harmonic progression if and only if its terms are the multiplicative inverse of an arithmetic progression that does not include a zero.

Defining Harmonic progression and the sum of the terms harmonic progression

The Harmonic Sequence is the term given to the reciprocal form of the Arithmetic Sequence, which uses numbers that are never going to equal 0. Furthermore, the total of such a sequence is referred to as the Harmonic Series.

II. Grey Areas in the Study of Harmonic Progression.

The relationship between the arithmetic progression and the harmonic progression.

Interestingly, we can form a harmonic progression from arithmetic progression by taking the reciprocals of the terms in the arithmetic regression. While we have a common ratio between two adjacent numbers in a geometric progression, in a harmonic progression, we have a common difference between two adjacent terns. A common raion in a geometric progression is denoted by r while a common difference in arithmetic series is denoted by d. in every sequence, the first term is denoted by a. this implies that we can get the next terms can be calculated given the common difference and or common ratio and one of the terms in the series. This can be done by a series of computations.

The nthterm in the arithmetic series

An arithmetic progression takes the general form; $a, a+d, a+2d, a+3d, \ldots$

The nth term is given by the formula $a_n = a+(n-1)d$, where a is the first term, d is the common difference and n is the nth term requested. This is an indication that the later progression, $2,4,6,8,\ldots$, is an arithmetic regression with 2 as both the common difference and the first term. A general rule suggests that; with two given terms of a series, if G.P, A.P, and H.P are specified as geometry, arithmetic, and harmonic progression, then, $A.P \geq G.P \geq H.P$. Similarly, $(A.P)(H.P) = (G.P)^2$

The nth term in a harmonic series

The harmonic series is the reciprocal of the arithmetic series. People will always confuse harmonic series and arithmetic series. Harmonic series take the form; $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots$By inspecting the trending and relating it to the arithmetic progression, one will realize that the nth term is given by the formula $a_n = \frac{1}{a+(n-1)d}$. Likewise, a and d are the first and nth requested respectively.According to (Yadav, 2008), The nth term of the Harmonic series = 1/ (nth term of the equivalent Arithmetic series). In this case, the first term of a harmonic progression obtained from the arithmetic progression, $2,4,6,8,\ldots$, will be, $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$ The first term is,

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$\frac{1}{2}$. However, taking the reciprocal does away with the common difference. This is one of the grey areas in the study of harmonic series. It is worth noting that a harmonic progression does not have a common difference.

Example1:

Given the harmonic series, $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots$, find the fourth term

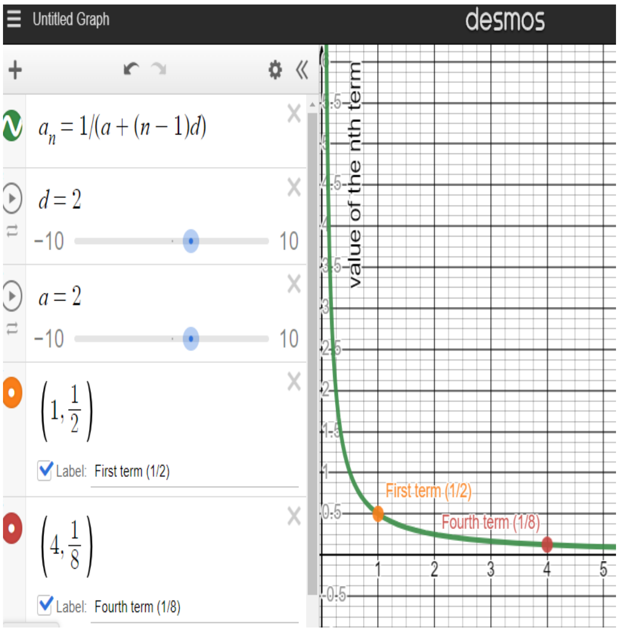
Solution

We use the formula; $a_n = \frac{1}{a+(n-1)d}$. Remember that d is the common difference for the corresponding arithmetic series $2, 4, and\ 6, \ldots$.

$$a_4 = \frac{1}{2+(4-1)2}$$
$$= \frac{1}{2+6}$$
$$= \frac{1}{8}$$

In any computation, the first term a and the common difference d, one must always use those in the corresponding arithmetic progression. In the above series, the graph of the harmonic series will be as in the table below.

A sample graph for the sum of the n[th] term in harmonic series



The fourth term is $1/8$ and the first term is $1/4$

The sum of n terms harmonic series.

In this case, we consider a general form of a harmonic progression, $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \cdots \frac{1}{a+(n-1)d}$. The sum will be given by the formula; $s\_n = (1/d) \times ln\left[\frac{2a+(2n-1)d}{2a-d}\right]$. Here, a is the first term of arithmetic progression, d is the common difference of arithmetic progression, and "ln" is the natural log. This formula is very different from that of the arithmetic series. The common difference d in an ammonic series remains the common difference of the corresponding arithmetic progression. For instance, the common difference in the harmonic series $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots,$ is 2 which is the common difference for the corresponding arithmetic progression, $2,4,6,8, \ldots,$.

Example. Compute the sum of the first four terms in the harmonic series $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots,$

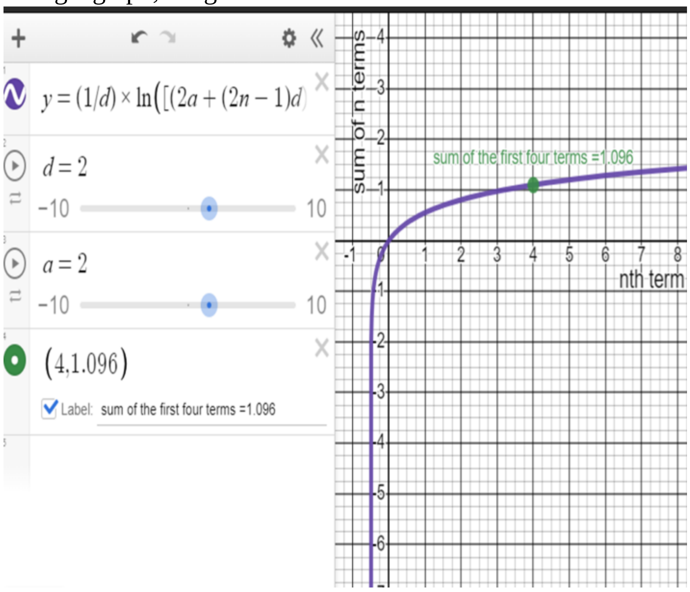
Solution

We compute this using the formula; $s_n = \left(\frac{1}{d}\right) \times \ ln\left[\frac{2a+(2n-1)d}{2a-d}\right]$

$$s_4 = \left(\frac{1}{2}\right) \times \ ln\left[\frac{2\times 2+(2\times 4-1)2}{2\times 2-2}\right]$$
$$= \frac{1}{2} \times Ln(9) = 1.0986$$

A sampe graph for the sum of n terms harmonic series

Using a graph, we get the same answer.



Understanding the difference between harmonic progression and arithmetic progression

Arithmetic and harmonic progressions are two different but almost similar and confusing. We have to find a way of defining the two series. Understanding arithmetic progression is very important in addressing and grasping key concepts on harmonic progression. Below is a table showing the key and noticeable differences.

Table 1: Key differences between arithmetic and harmonic progressions.

Criteria	Arithmetic progression	Harmonic progression
Calculation of we n^{th} term	We add fixed common differences d to the previous tern of the series. ;$a, a+d, a+2d, a+3d, \ldots$	We add fixed common differences d to the denominator of the previous tern of the series; $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots$
General form	It has the general form$a, a+d, a+2d, a+3d, \ldots$	Has the general form $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots$
nth tern formula	The n^{th} term is;$a_n = a + (n-1)d$	The n^{th} term is ; $a_n = \frac{1}{a+(n-1)d}$
The sum of nth tern formula	The formula for the sum of n terms is; $sn = \frac{1}{2b} n[2a + (n-1)d]$	The formula for the sum of n terms is; $sn = (1/d) \times ln\left[\frac{2a+(2n-1)d}{2a-d}\right]$
Example	2, 4, 6, 8…	H.P is the reciprocal of the arithmetic series. Example;$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$
Zero rule	Might contain zero	Does not contain zero

III. Impact of the Harmonic Progression on Life Situations.

Every mathematical concept learned is mos probably applicable in real-life situations. These are some of the things that many people do not know. Like in the case of arithmetic mathematics. Harmonic progression has been applied in the music industry in establishing theories on sound and the study of sound. To study the growth of the animals in a park, A simple harmonic progression is applied. For an instance, given that thee were 6 million deers in a park, increasing at a rate of 2000 per month. This information can be used to predict the number of deers in the park in the next 10 years.

It may come as a surprise to learn that the investigation of the harmonic sequence dates somewhere back to the sixth century when the Greek mathematician Pythagoras investigated the characteristics of the universe. The study of music was his first application for it. A harmonic series is an example of an infinite series, which does not have any limits and is characterized by the fact that the sum of progressively aspects tends to infinity.

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