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Graph Theory and Its Applications With A Focus on Assignment Problems

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***Abstract*— Graph theory is a branch of mathematics that deals with networks of points that are connected by lines. The beginnings of graph theory stemmed from recreational math problems and number and strategy games such as chess, but as a mathematical area of study, graph theory can be traced to famous mathematician Leonhard Euler. There was an old puzzle that concerned finding a path across every one of seven bridges that span a forked river around an island, but the path could only be created by crossing each bridge once. This is the famous Konigsberg bridge problem. Euler stated that no such path existed, and he was correct, although he could only prove it using the physical arrangement of the bridges and not in any mathematical terminology. His discovery of no such path paved the way for the first theorem of graph theory.**

A more specific focus in graph theory that we will be focusing on is graph coloring. A graph is properly colored when adjacent vertices of a graph connected by an edge are different colors. An easy way to ensure that adjacent vertices are colored differently is to give every vertex a unique color. However, this is a cumbersome method when it comes to coloring a large graph, and it is generally an inefficient method. This leads to the grand Four Color Theorem. The formal way of stating this theorem is that the chromatic number of a planar graph is no greater than four. A chromatic number is the least number of colors needed for the coloring of a graph. A planar graph is a dot-and-line diagram on a plane without any edges crossing except for at the established vertices. In other words, the Four Color Theorem just states that a planar graph can be colored using at most four colors. This is useful for things such as coloring a world map, because it would only take 4 colors to label every country. This theorem was originally conjectured in the 1850s by South African mathematician Francis Guthrie. In 1879 it was incorrectly proved by amateur mathematician Alfred Kempe but his proof held as valid until 1890 when British mathematician Percy Heawood found an error in Kempe's proof and ended up proving the five color theorem. The Four Color Theorem was official proved by American mathematicians Kenneth Appel and Wolfgang Haken in 1976 using a case by case analysis carried out by computer. This was the first major theorem to be proved using a computer, which is why it caused controversy in the math world since computers are not without flaws. Nevertheless, the proof is still upheld and there have been simpler proofs that shrank the number of configurations the computer has to go through, but no proof without the use of computers has been found yet.

***Index Terms*— graph theory, networks of points .**

I. Introduction

Assignment problems are used to model a wide range of applications in various disciplines. The underlying principle behind its formulation is the assignment of a task to something (i.e. workers) in order to increase efficiency. In this report, we shall look into a few of these applications namely:

1) Transportation and logistics
2) Job Assignments
3) Dating Websites

Transportation and Logistics

This is a very commonly used application of the assignment problem because it seeks minimize cost involved in the assignment of automobile (e.g. car, bus, train) to a destination(e. g warehouse, store) by using the shortest possible route to deliver goods. The aim of this type of modeling is not just to save transport cost, but also to save time to ensure goods are delivered to the end user in the most efficient manner. This is a very general application because it can be used to model inventory management, transportation networks (in this case the goods are the travelers) amongst others. It is an incredibly flexible application because it can be adapted to meet the needs of the adopter: some people might model their problem in a way that every transport system goes to one and only one destination while others might model theirs so two or more go to one location, and some might just randomize it in a way that whichever transport system is ready at a given time goes to the nearest point of need. It is all dependent on the users specific needs and constraints.

Job Assignments

As the name implies, this is an assignment application that involves assigning task(s) to worker(s) to ensure most efficient output. Think about any task you perform and imagine the task being a lot more complicated and on an even grander scale that requires division of labor amongst a certain group of people. How can this work be divided and who gets what task? These are the questions job assignment applications seek to answer based on the users needs. Some people might model the application in such a way that the worker with the best qualification for each task gets assigned that responsibility, some might model it based on hierarchy levels, some might model it based on years of experience while others might choose to do random assignments. Because of its generality, this can be used in any task/company/industry.

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Dating Website

This is another interesting application of the assignment problem in that it seeks to link every user of the website to another user with the end goal being ideal compatibility. This is made possible through users input on their wants and preferences, and an algorithm that goes on behind the scenes to provide other users who meet that criterion. This is a tricky use of the assignment problem because the suggested users by the website are only as good as the information they have provided and so it might not be the most efficient matching provided. However, like other applications it seeks to satisfy users wants and needs by providing likely satisfactory users in the least possible time, than it would take otherwise to search the website.

II. EXAMPLE PROBLEM

We will look at a two examples of assignment problems that can be modeled using matrices and graphs. We will also show that based on the required outcome, we can go more in depth to model assignments in a simple or more complex routes/directions. We will model these problems noting by:

Let C = [cij] be any n Å~n matrix in which cij is the cost of assigning worker i to job j . Let X = [xij] be the n Å~ n matrix where

xij = 1 if row i is assigned to column j (that is, worker i is assigned to job j) 0 otherwise

The assignment problem can then be expressed in terms of a function z as:

minimize z (X) =ΣΣCijXiji=1, j=1

subject to the constraints

j=1

Σxij = 1, for i = 1, 2. . ., n (1) … which guarantees each row entry is assigned 1 column entry

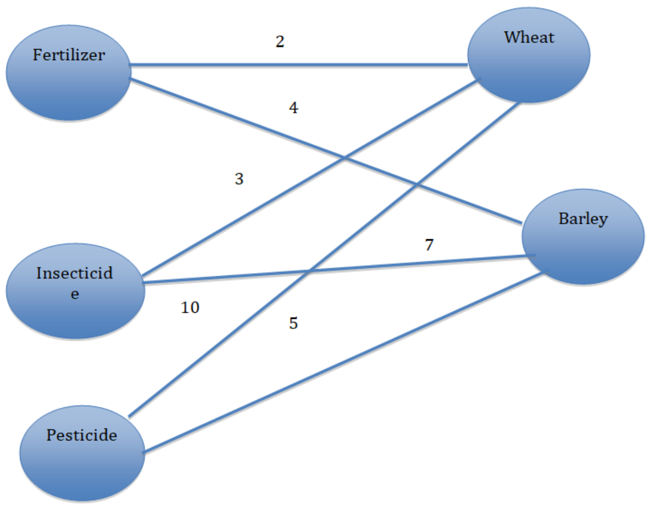
i=1

xij = 1, for j = 1, 2, . . ., n (2) … which guarantees each column entry is assigned 1 row entry (Rosen)

1) Farmland Problem

Ms. Victoria Lee Fitzgerald has a piece of farmland Lkm^2 to be planted with either wheat or barley or some combination of the two. Victoria has a limited amount of fertilizer, F kilograms, and insecticide, I kilograms and pesticide P kilograms. Every square kilometer of wheat requires F1 kilograms of fertilizer, I1 kilograms of insecticide, and P1 kilograms of pesticide while every square kilometer of barley requires F2 kilograms of fertilizer, I2 kilograms of insecticide and P2 kilograms of pesticide. Suppose Victoria wishes to minimize the cost associated with the combinations of fertilizer/wheat and pesticide for her wheat and barley farmland, which farmland would be best for her?

-	wheat	barley
Fertilizer	F1 = $2	F2 = $4
Insecticide	I1 = $3	I2 = $7
Pesticide	P1 = $10	P2 = $5



Above we have a simple graph that that has assignments Iw, Ib, Fw, Fb, Pw and Pb representing the insecticide/fertilizer/pesticide and wheat/barley combination. We can convert this into a matrix form to obtain

$$\begin{pmatrix} 2 & 4 \\ 3 & 7 \\ 10 & 5 \end{pmatrix}$$

We will solve this problem using permutations. We know that there are 2 possible outcomes:

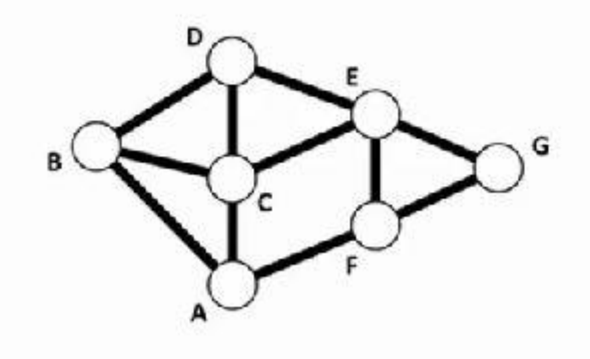
1) Fw + Iw + Pw = $2+ $3 + $10 = $15
2) Fb + Ib + Pb = $4+ $7 + $5 = $16

Using this simple example, we have modeled assignments of requirement for two different farmlands, and we have shown that the best combination would be for the wheat farmland (cost of $15) because it minimizes our total costs. We calculated the cost using calculator. This is a type of problem where we have used simplex graphs. There are so many other problems that can be modeled in more complex settings.

III. GRAPH COLORING

This example of graph coloring will focus on the greedy algorithm. A greedy algorithm is a mathematical process that looks for easy-to-implement solutions to complex problems by determining the next most beneficial step. For this problem, the greedy algorithm goes as follows:

1. Choose an order for the vertices
2. Choose a list of colors, also in some order
3. Color each vertex in order using the first available color on the list, being careful to make sure that no two adjacent vertices are the same color
4. Continue until each vertex is colored



So we can start with vertex A as yellow since nothing else is colored. Going on to vertex B, a different color must be used since it is adjacent to A, so it will be red. Vertex C must be blue since it is adjacent to vertices A and B. Vertex D needs to be a color other red or blue since it is adjacent to B and C, but since there is nothing adjacent to vertex D using yellow, and yellow is the lowest step in terms of colors, yellow can be used. Vertex E has to be something other than yellow or blue since it is adjacent to vertices D and C, so we can go back to red. Vertex F has adjacent vertices that are yellow and red, so the next best option is blue. The last vertex G must be yellow because vertex E is red and vertex F is blue

IV. DISCUSSION

This project enabled us to understand how graph theory is used not only to class examples but to real world application problems. Through this problem we modeled a simple linear programming optimization problem into a graphical format that we could simply solve using permutation. Although other means of obtaining an the optimal solution can be used for example AMPL modeling, we decided to stick completely to the mathematical formulation. It was an interesting project

V. SUMMARY OF CONTRIBUTION

Malcolm worked on the history of graph theory/assignment problems, the graph coloring aspect using the greedy algorithm example and put together the power point presentation. Temidayo worked on the applications of graph theory and assignment problems using platforms such as job assignments, dating websites etc She also worked on the farmland formulation and solution using permutation in the simple problem given. Finally she put most of the report together for submission.

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