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Member of the Metric Tensor g22 in Static Gravitational Field

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***Abstract*— This work is performed by the g_{22} term in the metric tensor and proves that its value $g_{22} = r^2 e^{2R/r}$,and the derivation is based on values $g_{11} = e^{2R/r}$ and according to Weinberg [6] (8,2,6) derives from the time member$g_{00} = -e^{-2R/r}$.**

In the paper, we also prove that metrics $g_{00} = -e^{-\frac{2R}{r}}, g_{11} = e^{2R/r}, g_{22} = r^2$ lead to a motion equation that does not give the correct values of light deflection and displacement of the planet's perihelion.

***Index Terms*— g_{22} term, Gravitational Field.**

I. Introduction

Exponential metrics have a time term $g_{00} = -e^{-2R/r}$, a radial term $g_{11} = e^{2R/r}$, and g_{22} in some authors it amounts to $g_{22} = r^2$.I will show that this term needs to look like in order $g_{22} = r^2 e^{2R/r}$for the equation of motion to give the correct deflection of light and the displacement of the perihelion of the planets.

Above g_{22} of the metric tensor member from Figure 2 [3], the observers in A (locally inertial system) and A' (non-inertial system with Gaussian coordinates) belong to the bases $\vec{\imath}_k$ *and* $\vec{\imath}_l$, respectively, for which:

$$\vec{\imath}_k \cdot \vec{\imath}_l = \mu_{kl} = \begin{Bmatrix} 1 & k = l = 1,\ 2,\ 3 \\ -1 & k = l = 0 \\ 0 & k \neq l \end{Bmatrix}; \vec{\imath}_k \cdot \vec{\imath}_l = g_{kl}$$

and the components of the vector $d\vec{r}$ are $(d|\vec{r}|, |\vec{r}|d\varphi_{g,} cdT)$ for the locally inertial system and $(dr, rd\varphi, cdt)$ for the non-inertial system.

The field is centrally symmetrical, static, and spatial construction does not depend on the direction but only on the position, which means $d|\vec{r}| = \alpha dr;\ |\vec{r}|d\varphi_{g,} = \alpha r d\varphi$ in the polar and $dx = \alpha dx';dy = \alpha dy'$ in the Cartesians system. The derivative in x_0 is:

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{\Delta x' \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x' \to 0} \frac{\alpha \Delta y'}{\alpha \Delta x'} = \lim_{\Delta x' \to 0} \frac{\Delta y'}{\Delta x'}$$

what from Figure 2 [3] means:

$$d\vec{r} = \vec{r_0} d|\vec{r}| + |\vec{r}|\overrightarrow{\Phi_0} d\varphi_g = \vec{r_0} \frac{\partial|\vec{r}|}{\partial r} dr + |\vec{r}|\vec{\Phi}_0 \frac{\partial \varphi_g}{\partial \varphi} d\varphi = \vec{e}_1 dr + \vec{e}_2 d\varphi$$

$$\text{and } (d\vec{r})_0 = \vec{r}dr + r\vec{\Phi}_0 d\varphi \Rightarrow (d\vec{r})_0 \times d\vec{r} = 0 \quad (1)$$

From (1) arises:

$$\frac{d|\vec{r}|}{dr} = \alpha = \sqrt{g_{11}} = \frac{|\vec{r}\,|^2 d\varphi_g}{\mathrm{r}d\varphi} = \frac{\sqrt{g_{22}}}{r}$$

$$g_{22} = r^2 g_{11}$$

From Figure 2 [3] obviously:

$$\Delta ABB' \sim \Delta ACC' \Rightarrow \frac{d|\vec{r}\,|}{dr} = \frac{|\vec{r}|d\varphi_g}{\mathrm{r}d\varphi}$$

Some authors popagate the metrics: $g_{00} = -e^{-\frac{2R}{r}},\ g_{11} = e^{2R/r}, g_{22} = r^2$
such as Vladimir Majernik Slovak Academy of Sciences which metric leads to equations of motion that do not give the correct deflections of light and displacements of the perihelion [8].
Namely, from Newton's equation of motion in an inertial system is:

$$\vec{F} = \frac{d\vec{p}}{\mathrm{dT}}, \vec{p} = \frac{md\vec{r}}{\mathrm{dT}} \quad (2)$$

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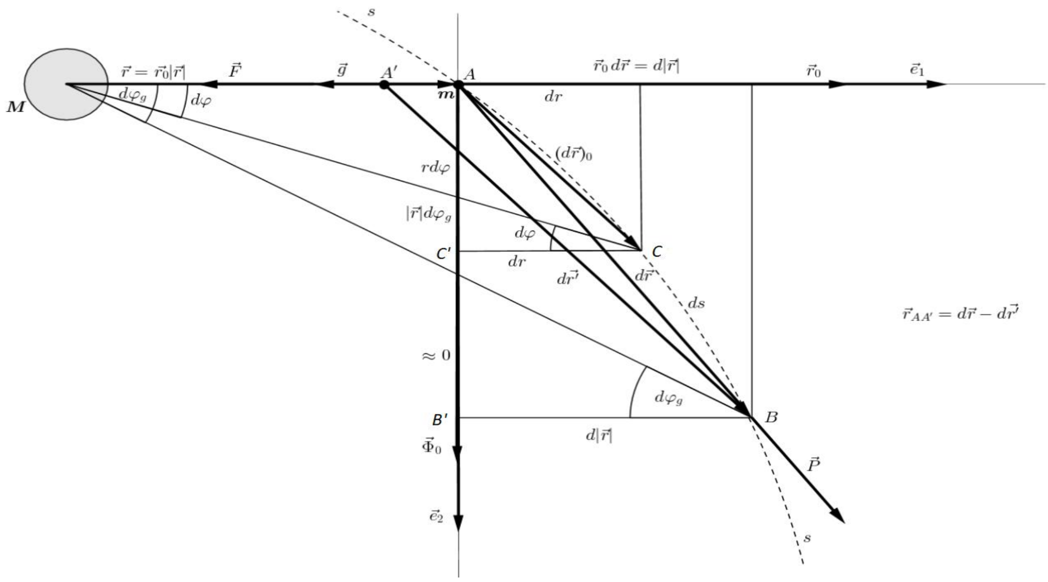

In non-inertial system with base $\vec{e}_\alpha$ (2) becomes:

$$F_\alpha = \vec{F}\cdot\vec{e}_\alpha = \vec{e}_\alpha\frac{d\vec{p}}{d\vec{t}} = \frac{dp_\alpha}{dt}\frac{dt}{dT} - \Gamma^\gamma_{\alpha\beta}p_\gamma\frac{dq^\beta}{dt}\frac{dt}{dT} = \left(\frac{dp_\alpha}{dt} - \Gamma^\gamma_{\alpha\beta}p_\gamma\dot{q}^\beta\right)\frac{dt}{dT}$$

In our metrics:

$$g_{22} = r^2e^{2R/r},\ \theta = \frac{\pi}{2} = const.,\ g_{11} = e^{2R/r},\ g_{00} = e^{-2R/r} \quad (3)$$

Christofel's symbols $\Gamma^\gamma_{\alpha\beta}$ different from 0 are:

$$\Gamma^1_{11} = -\frac{R}{r^2};\ \Gamma^2_{12} = \frac{1}{r}\left(1 - \frac{R}{r}\right);\ p_\varphi = mr^2\dot{\varphi}e^{\frac{3R}{r}};\ p_r = m\dot{r}e^{\frac{3R}{r}}$$

With $\vec{F}\cdot d\vec{r} = inv. \Longrightarrow F_1 = \frac{Rcm}{|\vec{r}|^3}\vec{r}\cdot\vec{r}_0 = \frac{Rcm}{|\vec{r}|^2}$ in inertial and $F_1 = -\frac{Rcm}{r^2}$ in noninertial system we arrive at the equation of motion in:

$$r = \frac{1}{\rho}$$

$$\ddot{\rho} + \rho = p^{-1} + 4R\rho p^{-1} + R(\rho^2 + \dot{\rho}^2);\ p^{-1} = \frac{Rc^2m_\infty{}^2}{p_\varphi{}^2};\ \mathrm{R} = \frac{GM_0}{c^2} \quad (4)$$

With metrics $g_{22} = r^2$ Kristofel's symbols are non-zero:

$$\Gamma^1_{11} = -\frac{R}{r^2};\ \Gamma^2_{12} = \frac{1}{r};\ p_\varphi = g_{22}m\frac{d\varphi}{dt}\frac{dt}{dT} = mr^2\dot{\varphi}e^{\frac{R}{r}};\ m = m_\infty e^{\frac{R}{r}}$$ leads to equation

$$\ddot{\rho} + \rho = p^{-1} - 2R\dot{\rho}^2 \quad (5)$$

(5) does not give the correct values of light deflection and perihelion displacement of the planets.

II. Conclusion

Metrics $g_{00} = e^{-2R/r}$, $g_{11} = e^{2R/r}$ and $g_{22} = r^2 e^{2R/r}$ with $\theta = \frac{\pi}{2} = const.$ provides motion equations that correctly describe the displacement of the planet's perihels and the deflection of light and leads to a wave equation that is bicubic in the problem of its own values.

The space in which quantum equations apply has a granular dimension structure of Planck length, and therefore it is worth using discrete mathematics to calculate the value of physical sizes.

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