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Einstein Field Equations with Exponential Metric

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***Abstract*— In this paper, a procedure for determining Einstein's equation withexponential metric is shown in the case of static, centrally symmetric, gravitational field. The procedure has been conducted by introducing a covariance four-vector forstatic, gravitational field that has its values A_μ (0,0,0,Φ). A scalar potential Φ is determined for the relativistic case of how body mass depends on its position in a gravitational field. Exponential metric in default with a metric tensor g_(μν)has been used to determine the Einstein tensor G_(μν). The energy-momentum tensor was obtained using the flat space metric.**

***Index Terms*— Einstein's equation, gravitational field, Exponential Metric.**

I. Introduction

Using ananalogy between electromagnetic fields and anelectromagnetic potential, we can show a gravitational field throughagravitational potential.

The gravitational potential will be expressed through covariance four-vector $A_{\mu} = \left(A_0, \vec{A}\right)$, where $A_0 =$ Φisscalar potential, whereas $\vec{A}$ is vector potential expressed through components of the metric tensor, i.e.

$A_k = \frac{g_{0k}}{\sqrt{-g_{00}}}$, $k = 1, 2, 3$ [1].

We will consider the case of a body motion of mass *m* in static gravitational field with source mass *M*, whereby $M \gg m$.

Vector potential for static gravitational field will be: $\vec{A} = 0$, so $A_{\mu} = (\Phi, 0)$.

To determine a scalar potential Φ, we will consider a body motion of mass *m* in a gravitational field:

$$\vec{g} = \frac{Rc^2}{|\vec{r}|^3}\vec{r} \qquad (1)$$

where $R = \frac{GM}{c^2}$ is a gravitational radius.

The gravitational force that acts on the body of mass *m* will be:

$$\vec{F} = -\frac{Rc^2}{|\vec{r}|^3} m\vec{r}. \qquad (2)$$

The change of the gravitational potential energy equals the work of force $\vec{F}$ in a gravitational field, i.e.:

$$dU = -\vec{F}d\vec{r} = \frac{Rc^2 m}{|\vec{r}|^3}\vec{r}\cdot d\vec{r} \qquad (3)$$

Regarding the relativistic case, thechange relation of kinaesthetic energy is valid [2]:

$$dE_k = -dU = c^2 dm \qquad (4)$$

From the equations (3) and (4) we obtain differential equations in the following form:

$$\frac{dm}{m} = -\frac{R}{|\vec{r}|^2}d\vec{r} \qquad (5)$$

This equation has a solution

$$m = m_{\infty} e^{R/r} \qquad (6)$$

where $m = m_{\infty}$ when $|\vec{r}| = \infty$ [3].

The equation (6) shows mass's dependence of its position in a gravitational field.

Including the mass (6) in the equation (3) we obtain gravitational potential energy in the following form:

$$U = -m_{\infty}c^2\left(e^{R/r} - 1\right) \qquad (7)$$

When using the equation (7),a scalar potential Φ is

$$\Phi = \frac{U}{m_{\infty}} = -c^2\left(e^{R/r} - 1\right) \qquad (8)$$

II. Gravitational Field Tensors

The scalar potential Φ, specified by the equation (8), is used to determine gravitational field tensor

$$F_{\mu\nu} = \frac{\partial A_{\mu}}{\partial x^{\nu}} - \frac{\partial A_{\nu}}{\partial x^{\mu}} \qquad (9)$$

whereas $F_{\mu\nu} = 0$, $\forall\, \mu, \nu = 1, 2, 3$.

The only components different from zero will be $F_{0\mu} = -F_{\mu 0}$.

In the polar coordinate system, the following relations are also valid:

$$F_{01} = -F_{10} = \frac{\partial\Phi}{\partial r}$$

$$\frac{\partial\Phi}{\partial\vartheta} = \frac{\partial\Phi}{\partial\varphi} = 0$$

Applying the listed relations from the equation (9), we obtain gravitational field tensor in the following form:

$$F_{\mu\nu} = \frac{Rc^2}{r^2}e^{2R/r}\cdot\begin{vmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \qquad (10)$$

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We determine gravitational field energy tensor analogous to electromagnetic field energy tensor according to the following relation:

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\lambda}F_{\lambda\nu} + \frac{\delta_{\mu\nu}}{4}F_{\alpha\beta}F^{\alpha\beta}\right) \quad (11)$$

We will determine the energy tensor $T_{\mu\nu}$ (11) for flat space using the metric tensor

$$g_{\mu\nu} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \quad (12)$$

so, we obtain

$$T_{\mu}^{\nu} = \frac{c^4R^2}{8\pi r^4}\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} e^{2R/r} \quad (13)$$

Now we will determine the Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \qquad (14)$$

By applyingexponential metric, we use components of the metric tensor $g_{\mu\nu}$ in the polar coordinate system $x^{\mu} = (ct, r, \vartheta, \varphi)$ [4, 5], i.e.

$$g_{00} = -e^{-2R/r}$$

$$g_{11} = e^{2R/r}$$

$$g_{22} = r^2 e^{2R/r}$$

$$\vartheta = \frac{\pi}{2} \qquad (15)$$

$$g_{33} = r^2 \sin^2\vartheta \cdot e^{2R/r}$$

Using components of the metric tensor (15) we can determine Christoffel symbols according to the relation

$$\Gamma_{\alpha\beta}^{\mu} = \frac{1}{2}g^{\mu\delta}\left[\frac{\partial g_{\alpha\delta}}{\partial x^{\beta}} + \frac{\partial g_{\beta\delta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\delta}}\right] \quad (16)$$

while Ricci tensor is determined by the relation

$$R_{\mu\nu} = \frac{\partial \Gamma_{\mu\nu}^{\lambda}}{\partial x^{\lambda}} - \frac{\partial \Gamma_{\mu\lambda}^{\lambda}}{\partial x^{\nu}} + \Gamma_{\mu\nu}^{\lambda}\Gamma_{\lambda\chi}^{\chi} - \Gamma_{\mu\lambda}^{\chi}\Gamma_{\nu\chi}^{\lambda} \quad (17)$$

$$R_{00} = R_{22} = R_{33} = 0$$

$$R_1^1 = -\frac{2R^2}{r^4}c^{-2R/r} = R$$

$$R_{\mu\nu} = 0 \quad \forall\, \mu \neq \nu$$

By applying the relations (15), (16) and (17) we obtain components of the Einstein tensor $G_{\mu\nu}$, i.e.

$$G_0^0 = R_{00} - \frac{1}{2}R = \frac{R^2}{r^4}e^{-2R/r}$$

$$G_1^1 = R_{11} - \frac{1}{2}R = -\frac{R^2}{r^4}e^{-2R/r}$$

$$G_2^2 = R_{22} - \frac{1}{2}R = \frac{R^2}{r^4}e^{-2R/r} \qquad (18)$$

$$G_3^3 = R_{33} - \frac{1}{2}R = \frac{R^2}{r^4}e^{-2R/r}$$

$$G_{\mu\nu} = 0 \quad \forall\ \mu \neq \nu$$

Comparing the values of the Einstein tensor $G_{\mu\nu}$ (18) and the energy tensor $T_{\mu\nu}$ (13) we come to the conclusion that exponential metric with components of the metric tensor $g_{\mu\nu}$ (15) satisfies Einstein's equation in the first approximation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (19)$$

In the natural system of units, i.e. $c = G = \hbar = 1$, we write the equation (19) in this form

$$G_\mu^\nu = \frac{R^2}{r^4} e^{2R/r} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (20)$$

III. Conclusion

A procedure analogous to the procedure for the electromagnetic field has been conducted to determine Einstein's equation forstatic, centrally symmetric, gravitational field. The Einstein tensor $G_{\mu\nu}$ was obtained using exponential metric given by the metric tensor $g_{\mu\nu}$ in the polar coordinate system. Determination of the energy-momentum tensor was performed using the flat space metric. The given result shows that exponential metric satisfies Einstein's equation in the first approximation alongside the energy-momentum tensor in a flat space.

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