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Metric Topology and Visual Perception of Image

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Abstract— **In mathematics, a metric space is a set for which distances between all members of the set are defined. Those distances, taken together, are called a metric on the set. A metric on a space induces topological properties like open and closed sets, which lead to the study of more abstract topological spaces.The most familiar metric space is 3-dimensional Euclidean space. In fact, a "metric" is the generalization of the Euclidean metric arising from the four long-known properties of the Euclidean distance. The Euclidean metric defines the distance between two points as the length of the straight line segment connecting them. Other metric spaces occur for example in elliptic geometry and hyperbolic geometry, where distance on a sphere measured by angle is a metric, and the hyperboloid model of hyperbolic geometry is used by special relativity as a metric space of velocities.**

Index Terms— **Metric digital image, Topology digital image.**

I. Introduction

Digital topology deals with properties and features of two-dimensional (2D) or three-dimensional (3D) digital images that correspond to topological properties (e.g., connectedness) or topological features (e.g., boundaries) of objects.

Concepts and results of digital topology are used to specify and justify important (low-level) image analysis algorithms, including algorithms for thinning, border or surface tracing, counting of components or tunnels, or region-filling.

A digital image is a numeric representation, normally binary, of a two-dimensional image. Depending on whether the image resolution is fixed, it may be of vector or raster type. By itself, the term "digital image" usually refers to raster images or bitmapped images (as opposed to vector images).

II. Metric and Topology of Digital Images

A digital image consists of picture elements with finite size these pixels carry information about the brightness of a particular location in the image. Usually (and we assume this hereafter) pixels are arranged into a rectangular sampling grid. Such a digital image is represented by a two-dimensional matrix whose elements are integer numbers corresponding to the quantization levels in the brightness scale.Some intuitively clear properties of continuous images have no straightforward analogy in the domain of digital images [pavlidis 77, Ballard and brown 82]. **Distance** is an important example the distance between points with co-ordinates (i, j) and (h, k) may be defined in several ways; the **Euclidean distance** De known from classical geometry and several everyday experiences is defined by

$$De\ [(i, j), (h, k)] \ = \sqrt{(i-h)^2 + (j-k)^2}$$

The advantage of the Euclidean distance is the fact that it is intuitively obvious. The disadvantages are costly calculation due to the square root, and its non-integer value.

The distance between two points can be also be expressed as the minimum number of elementary steps in the digital grid which are needed to move from the starting points to the end points. If only horizontal and vertical moves are allowed, the distance because of the analogy with the distance between two locations in a city with a rectangular grid of streets and closed block of the houses.

$$D4\ [(i, j), (h, k)] = |\, i - h \,| + |\, j - k \,|$$

If moves in diagonal directions are not allowed in the digitization grid, we obtain the distance D8, often called 'chessboard' distance. The distance D8 is equal to the number of moves of the king on the chessboard from one part to

$$D8\ [(i, j), (h, k)] = max\,\{|\, i - h \,|, |\, j - k \,|\}|$$

Any of these metrics may be used as the basis of **chamfering**, in which the distance of pixels from some images subset (perhaps describing some feature) is generated. The resulting 'image' has pixel values of 0 for elements of the relevant subset, low values for close pixels, and then high values for pixels remote from it the appearance of this array gives the name to the technique. Chamfering is of value in **chamfer matching**. The algorithm is derived from one based on a simplification of the Euclidean metric.

AL	**AL**	**BR**
AL	**AL** **BR**	**BR**
AL	**BR**	**BR**

Figure 1 *Pixel neighborhood used in Chamfering-pixel P is the central one*

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A. Algorithm Chamfering

1. To chamfering a subset S of an image of dimension $M \times N$ with respect to a distance metric D, where D is one of $D4$ or $D8$, construct an $M \times N$ array F with elements corresponding to the set S to O, and each other elements to infinity

2. Pass through the image row-by-row, from bottom to top and right to left. For each neighboring pixel below and to right, illustrate in fig 1 by the set Al, set

$$F(p) = \min [F(p), D(p, q) + F(q)]$$

3. Pass through the image row-by-row, from bottom to top and right to left. For each neighboring pixel below and to the right, illustrated in Figure by the set BR, set

$$F(p) = \min [F(p), D(p, q) + F(q)]$$

4. The array F now holds a chamfer of the subset S.

This algorithm needs obvious adjustment at the image boundaries, where the sets AL and BR are truncated.

Pixel **adjacency** is another important concept in digital images. Any two pixels are called **4-neighbour** if they have distance $D4 = 1$ from each other. Analogously, **8-neighbours** and two pixels with $D8 = 1$. *4*-neighbour and 8-neighbour are illustrated.

It will become necessary to consider important sets consisting of several adjacent pixels **regions**. For those familiar with set theory, we can simply say that a region is a contiguous set. More descriptively, we can define a path from pixel P to pixel Q as a sequence of points $A, A2 \ldots An$, where $A1 = P$, $An = Q$, and $Ai +1$ is a neighbor of Ai, $i = 1 \ldots n - 1$; then a **region** is a set of pixels in which there is a path between any pair of its pixels, all of whose pixels also belong to the set.

If there is a path between two pixels in the images, these pixels are called contiguous alternatively; we can say that a region is the set of the pixels where each of pair of pixels in the set is contiguous. The relation to be contiguous is reflexive, symmetric, and transitive and therefore defines a decomposition of the set (in our case image) into equivalence classes (regions).

Assume that Ri is disjoint regions in the image which were created by the relation to be contiguous, and further assume (to avoid special cases) that these regions do not touch the images limits (meaning the rows or columns in the images matrix with minimum and maximum indices). Let region R be the union of all regions Ri; it then becomes sensible to define a set Rc that is the set complement of region R with respect to the image. The subset of Rc, which is contigious with the image limits, is called **background**, and the rest of the complement Rc is called **holes4**. If there are no holes in a region we call it a simply **contiguous** region. A region with holes is called **multiply contiguous.**

Note that the concept of region uses only the property to be contiguous. Secondary properties can be attached to region uses, which originate in image data interpretation. It is common to call some regions in the image **objects;** a process, which determines which regions in an image corresponding to objects in the world, is part of images **segmentation.**

The brightness of a pixel is a very simple property, which can be used to find objects in some images; if, for example, a pixel is darker than some predefined value (threshold), then it belongs to the object. All such points, which are also contiguous, constitute one object. A hole consists of points which do not belong to the object and are surrounded by the object, and all other points constitute the background.

An example is the black printed text on this white sheet of the paper, in which individual letters are objects. White areas surrounded by the letters are holes, for example, the area inside a letter ‘o’. Other white parts of the paper are background.

These neighborhood and contiguity definitions on the square grid create some paradoxes. Two digital line segments with 45° slope. If 4-connectivity is used, the lines are not contiguous at each of their points. An ever worse conflict with intuitive understanding of line properties is also illustrated; two perpendicular lines do intersect in one case (upper right intersection) and do not intersect in another case (lower left), as they do not have any common point (i.e. their intersection is empty).

It is known from Euclidean geometry that each closed curve (e.g., a circle) divides the plane into two non-contiguous regions. If images are digitized in a square grid using 8-connectivity, we can draw a line from the inner part of a closed curve into the outer part which does not intersect the curve , this implies that the inner and outer parts of the curve constitute only one region because all pixels of the line belongs to only one region. This is another paradox.

One possible solution to contiguity paradoxes is to treat using 4-neighborhoods and background using 8-neighborhoods (or vice verse). More exact treatment of digital images.

Paradoxes and there solution for binary images and images with more brightness levels can be found in [pavlidis 77, horn 86].

These problems are typical on square grid a hexagonal grid, however solves many of them. Any point in the hexagonal raster as well as for instant it is difficult to express a Fourier transformer on it.

An alternative approach to the connectivity problems is to be used discrete topology based on cell complexes [Kovalevsky 89]. This approach develops a complete strand of images encoding and segmentation that deals with many issues we shall come to later, such as the representation of boundaries and regions. The idea, first proposed by Riemann in the nineteen century, consider families of sets of different dimensions; points, which are 0-dimensional structure (such as pixel arrays), which permits the removal of the paradoxes we have seen.

For reasons of simplicity and ease of processing, most digitizing devices use square grid despite the stated drawbacks.

The **border** of a region is another important concept in image analysis. The border of a region R is the set of pixels within the region that have one or more neighbors outside R. the definition corresponds to an intuitive understanding of the border as a set of points at the limit of the region. This

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definition of borders is sometimes referred to as **inner border** to distinguish it from the **outer border,** which is the border of the background (i.e., its complement) of the region.

An **edge** is a further concept. This is a local property of a pixel and its immediate neighborhood is a vector given by a magnitude and direction. Image with many brightness levels are used for edge computation, and the gradient of the image function is used to compute edges. The edges direction is perpendicular to the gradient direction which points in the direction of image function growth.

Note that there is a difference between border and edge. The border is global concept related to a region, while edge expresses local properties of an image function. The border and edge are related as well. One possibility for finding boundaries is chaining the significant edges (points with high gradient of the image function).

The edge property is attached to one pixel and its neighborhood sometimes it is of advantage to assess properties between the pair of neighboring pixels, and the concept of the **crack edge** comes from this idea. Four crack edges are attached to each pixel, which are defined by its relation to its 4-neighborhoods. The direction of the crack edges is that of increasing brightness of the relevant pair of pixels..

B. Topological properties

Topological properties of images are invariant to rubber sheet transformations. Imagine a small balloon with an object painted on it; topological properties of the object are those, which are invariant to the arbitrary stretching of the rubber sheet. Stretching does not change contiguity to the object parts and does not change the number of holes in regions. One such image property is the **Euler-poincare characteristic,** defined as the difference between the number of region and the number of holes in them.

A convex hull is a concept used to describe topological properties of objects. The convex hull is the smallest region which contains the object, such that any two points of the region can be connected by the straight line, all points of which belongs to the region for example, consider an object whose shape resembles a letter 'R'. Imagine a rubber band pulled around the object; the shape of the rubber band provides the convex hull of the object.

An object with non-regular shape can be represented by a collection of its topological components. The set inside the convex hull, which does not belong to an object, is called the **deficit of convexity;** this can be split into two subsets. First, **lakes** are fully surrounded by the object; and second, **bays** are contiguous with the border of the convex hull of the object.

The convex hull, lakes and bays are sometimes used for object description.

III. HISTOGRAMS

The brightness histogram *hf (z)* of an image provides the frequency of the brightness value *z* in the image the histogram of the image with *L* gray levels is represented by a one-dimensional array with *L* elements.

A. Algorithm : computing the brightness histogram

1. 1. Assign zero value to all elements of the array *hf*
2. 2. For all pixels *(x, y)* of the image *f*, increment *hf [f (x, y)]* by 1.

Recalling that an image can be analyzed as the realization of a stochastic process, we might want to find a first order density function *p1 (z, x, y)* to indicate function *p (x, y)* brightness *z*. if the position pixel is not interest, we obtain a density function *p1 (z)*, and the brightness histogram is its estimate.

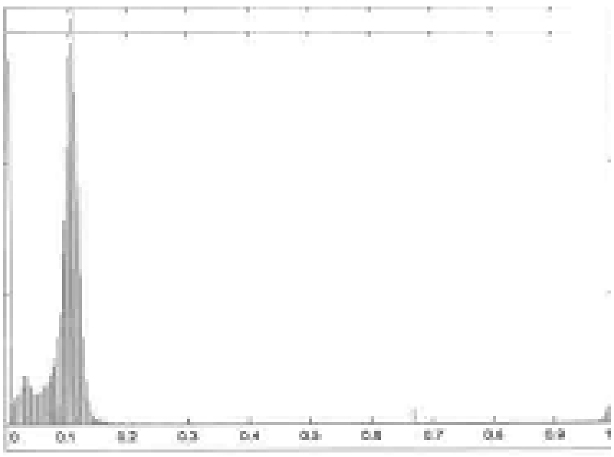

Figure 2 *Histogram of the image*

The histogram is often displayed as a bar graph. The histogram of the image in Figure 2 .

The histogram is usually the only global information about the image, which is available. It is used when finding optimal illumination conditions for capturing an image, gray scale transformations, and image segmentation to objects and background. Note that one histogram may correspond to several images. For instance, change of object position on a constant background does not affect the histogram.

The histogram of a digital image typically has many local minima and maxima, which may complicate its further processing. This problem can be avoided by local smoothing of the histogram; this may be done, for example, using local averaging of neighboring histogram elements as the base, so that a new histogram *h`f (z)* is calculated according to

$$h'f(z) = 1/2k+1\sum_{i=-k}^{k} hf(z+i).$$

Where **K* is a constant representing the size of the neighborhood used for smoothing. This algorithm would need some boundary adjustment, and carries no guarantee of removing all local minima. Other techniques for smoothing exist, notably gauss Ian blurring; in the case of a histogram, this would be a one dimensional simplification of the 2D gauss Ian blur.

IV. VISUAL PERCEPTION OF THE IMAGE

Anyone who creates or uses algorithms or devices for digital image processing should take into account the principles of human images is to be analyzed by a human the information should be expressed using variables which are easy to perceive; these are psycho physical parameters such as contrast, border, shape, texture, color, etc. Humans will find objects in image only if they may be distinguished effortlessly from the background. A detailed description of the principles of human perception can be found in [corn sweet 70, Winston 75, marr 82, Levine 85]. Human perception of images provokes. Many illusions, the understanding of which provides valuable clues about visual mechanisms. Some of the better known illusion will be mentioned here the topics is covered exhaustively from the point of view of computer vision in [Frisby 79].

The situation would be relatively easy if the human visual system had a linear response to composite input stimuli i.e., a simple sum of individual stimuli. A decrease of some stimulus, e.g., area of the object in the image, could be compensated by its intensity, contrast, or duration. In fact, the sensitivity of human sense is roughly logarithmically proportional to the intensity of an input signal. In this case, after an initial logarithmic transformation response to composite stimuli can be treated as linear.

A. Contrast

Contrast is the local change in brightness and is defined as the ratio between average brightness of an object and the background brightness. The human eye is logarithmically sensitive to brightness, implying that for the same perception, higher brightness requires higher contrast.

Apparent brightness depends very much on the brightness of the local background; this effect is called conditional contrast. Figure 2.14 illustrate this with two small squares of the same brightness on a dark and light background. Humans perceive the brightness of the small square as different.

B. Acuity

Acuity is the ability of defects details in the image. The human eye is less sensitive to slow and fast changes in brightness in the image plane but is more sensitive to intermediate changes. Acuity also decreases with increasing distance from optical axis,

Resolution in an image is firmly bounded by the resolution ability of the human eye; there no sense in representing the visual information with higher resolution than that of the viewer, resolution in optics is defined as the inverse value of the maximum viewing angle between the viewer and two proximate points which cannot distinguish, and so fuse together.

Human vision has the best resolution for objects, which are at a distance of about 250mm from an eye under illumination of about 500 lux; a 60w bulb from a distance of 400mm provides this illumination. Under these conditions the distance between two distinguishable points is approximately 1.16mm

C. Object Order

Object borders carry a lot of information [marr 82]. Boundaries of objects and simple patterns such as blobs or lines enable adaptation effects similar to conditional contrast, mentioned above. The Ebbinghaus illusion is a well-known example two circles of the same diameter in the center of images appear to have different diameters (Figure: 3).

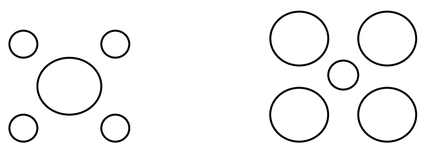

Figure 3 *The Ebbinghaus illusion*

D. Color

Color is very important for perception, since under normal illumination conditions the human eye is more sensitive to color than to brightness. Color quantization and representation are described.

Color can be expressed as a combination of the three basic components, red, green, and blue (RGB), but color **perception** is better expressed in the alternative H I S co-ordinate system.

Color perception is burdened with similar adaptation illusions as other psychophysical quantities

V. Conclusion

An image might be degraded during capture, transmission, or processing, and measures of image quality can be used to assess the degree of degradation. The quality required naturally depends on the purpose for which an image is used.

Methods for assessing image quality can be divided into two categories: subjective and objective. Subjective methods are often used in television technology, where the ultimate criterion is the perception of a selected group of professional and law viewers. They appraise an image according to a list of criteria and give appropriate marks. Details about subjective methods may be found in [pratt 78].

Objective quantitative methods measuring image quality are more interesting for our purposes. Ideally such a method also provides a good subjective test, and is easy to apply; we might then use it a criterion in parameter optimization. The quality of the image $f\ (x,\ y)$ is usually estimated by comparison with a known reference image $g\ (x, y)$ [rosenfeld and kak 82]. A synthesized image is often used for this purpose.

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AUTHOR'S PROFILE

Dr Mir Mohammad Azad was born in Village – Korer Betka; Post Office: Mirrer Betka; Police Station - Tangail; District - Tangail, Bangladesh on 10th October, 1982.He received PhD in Computer Science, 2008 from Golden State University, Master of Computer Application, 2006 from Bharath Institute of Higher Education and Research Deemed University (Bharath University) and Bachelor of Computer Application, 2004, Bangalore University, India. He also received Bachelor of Law (LL.B) from National University of Bangladesh. He was working as a lecturer and head of computer science in various colleges in Bangalore and also worked as an Assistant professor and Vice Principal in different colleges in Bangalore during the year (2005-2009).He worked as an Assistant Professor and Head of CSE & CSIT at Shanto Mariam university of Creative Technology (2010-2014). He is having 27 publications in international journal in various countries like UK, USA, FRANCE, KOREA, PAKISTAN, INDIA, GERMAN, and JAPAN. At present he is working as an Associate Professor, Department of Computer Science and Engineering, Department Computer Science and Information Technology in Shanto Mariam university of Creative Technology, Uttara, Dhaka, Bangladesh. His areas of interest include Computer Architecture, Architecture, E-commerce, Digital Image processing, Computer Network, Wireless communication, MIS and Law.

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Muhammad Aminuzzaman was born in Dhaka, Bangladesh on 11th March 1972. He completed his Master of Commerce (Marketing) in 1995 and Bachelor of Commerce (Honors in Marketing) in 1994 from the University of Dhaka, Bangladesh. He has completed a three-year course on photography from Pathshala, The South Asian Institute of Photography. His photography has been exhibited in Bangladesh and Malaysia. His images were published in international magazines like the Time Journal of Photography, The Saudi Aramco World and the book, Our World of Water by Oxfam GB. He also contributed to five documentaries filmed in Bangladesh and in India for the TVE (Television for the Environment), which were broadcasted in BBC's 'Earth Report'. He worked as a professional photographer and multimedia journalist at Drik Picture Library Limited from 2002-2015.He worked as a Senior lecturer at Green University of Bangladesh from 2014-2018. At present he is working as a Senior Lecturer, Department of Media Studies and Journalism, in University of Liberal Arts Bangladesh. His areas of interest include Photography, Digital Image processing, Communication, Marketing and Film.