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Unification of SR and GR and Four Fundamental Interactions in RAF Theory

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***Abstract*- As we know, there exists requirement for unification of Special Relativity (SR) and General Relativity (GR) into one self-consistent theory. On the other side, there also exists requirement for unification of four fundamental interactions in the standard four dimensions (4D). Recently, it has been developed a new Relativistic Alpha Field Theory (RAFT or RAF theory) that can be used for the mentioned unifications. Namely, in RAF theory it has been introduced an alpha field as the function of two dimensionless field parameters α and α'. These parameters are the functions of the normalized potential energy of a particle in an alpha field. If there no potentials, then field parameters α and α' become equal to unity, and all items in GR are transformed into the related items in SR. Thus, RAF theory unifies SR and GR into one self-consistent theory. Further, the fact that field parameters α and α' are the functions of the normalized potential energy of a particle in an alpha field opens ability to unify all fundamental interactions in the standard four dimensions (4D). Here it has been shown that RAF theory is the adequate candidate for the unification of the four fundamental interactions in standard 4D, because it extends the applications of GR to the extremely strong gravitational field, including of the Planck's scale.**

***Index Terms*- Relativistic Alpha Field Theory (RAFT), Determination of field parameters, Unification of SR and GR, Unification of four fundamental forces**

I. Introduction

In the today's physics we have three self-consistent, relatively successful, theories: Special Relativity (SR), General Relativity (GR) and Quantum Mechanics (QM). Meanwhile, there exists requirement for unification of those three theories into one self-consistent theory. The first step could be the unification of SR and GR into one self-consistent theory. On the other side, there also exists requirement for unification of four fundamental interactions in the standard four dimensions (4D).

For unification of gravity interaction with the other tree fundamental interactions (weak, strong and electromagnetic), one can use the following two possibilities [1-6]: a) trying to describe gravity as a gauge theory, or b) trying to describe gauge theories as gravity. The first possibility (a) has attracted a lot of attention, but because of the known difficulties, this approach set gravity apart from the standard gauge theories. The second possibility (b) is much more radical. The initial idea has been proposed by Kaluza-Klein theory [7, 8], which today has many variations [9-14], and takes the place in the modern theories like high energy physics (supergravity [15-17] and string theories [18-28]). These theories use five or more extra dimensions with the related dimensional reduction to the four dimensions. Meanwhile, we do not know the answers to some questions like: can we take the extra dimensions as a real, or as mathematical devices?

Recently, it has been developed a new Relativistic Alpha Field Theory (RAFT) [29-31] that could be used for the unification of SR end GR, as well as, for unification of four fundamental interactions in the standard four dimensions (4D). This unification is based on the geometric approach. Namely, in RAF theory it has been introduced an alpha field as the function of two dimensionless field parameters α and α'. Those parameters are the functions of the normalized potential energy of a particle in an alpha field. If there no potentials, then field parameters α and α' become equal to unity, and all items in GR are transformed into related items in SR. On that way, RAF theory unifies SR and GR into one self-consistent theory.

Further, the fact that field parameters α and α' are dimensionless functions of the normalized potential energy of a particle in an alpha field opens ability to unify all fundamental interactions in the standard four dimensions (4D). This unification is based on the normalized potential energy level. We know that there exists successful unification of three fundamental interactions (electromagnetic, weak and strong). But, problem is with unification of the mentioned interaction with gravitational interaction. As it is well known [1-6], GR cannot be applied to the extremely strong gravitational field including Planck's scale, because of the related singularity. Here we present that Relativistic Alpha Field (RAF) theory can be used for the unification of all fundamental interactions in the standard four dimensions (4D). Namely, RAF theory extends the applications of GR to the extremely strong gravitational field, including the Planck's scale [29-31]. Therefore, RAF theory is the adequate candidate for the mentioned unification of four fundamental forces in standard four dimensions (4D). This is the consequence of the following predictions of RAF theory: a) no a singularity at the Schwarzschild radius and b) there exists a minimal radius at r = $(GM/2c^2)$ that prevents singularity at r = 0, i.e. the nature protects itself. Predictions a) and b) are presented in the second part of this theory [30]. Since, Quantum Mechanics (QM) is also regular at the Planck's scale, the possibility of the future unification of GR and QM is also open. One possibility of this unification is presented in the article Quantum Gravity in Relativistic Alpha Field Theory (QG in RAFT) [32].

In order to determine the field parameters α and α', we started with the derivation of the relative velocity of a particle in an alpha field, v_α. This relative velocity is derived from the line element in an alpha field given by the nondiagonal form with the Riemannian metrics. Thus, the relative velocity of a particle in an alpha field, v_α, is described as the function of the field parameters α and α' and a particle velocity v in

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the total vacuum (without any potential field). This structure of the relative velocity v_α directly connects the line elements of the SR and GR. Namely, in the case of the total vacuum (without any potential field), field parameters α and α' become equal to unity and, consequently, the relative velocity v_α becomes equal to the particle velocity v in the total vacuum. This is the transformation of the line element from the GR to the SR.

This paper is organized as follows. In Sec. II, we show derivation of the relative velocity of a particle in an alpha field v_α as the function of the field parameters α and α'. Derivation of the field parameters α and α' in a general form, as the function of the normalized potential energy U is presented in Sec. III. Solution of field parameters in unified field has been described in sec. IV. Solution of field parameters in gravitational field is present in Sec. V. The solution of the field parameters α and α' in the unified electrical and gravitational field is pointed out in Sec. VI. Finally, the related conclusion and the reference list are presented in Sec. VII and Sec. VIII, respectively.

II. Derivation of Relative Velocity v_α

The basic problem of this paper is to determine the field parameters α and α' of a particle in the unified four fundamental fields. The first step in this determination is the derivation of the relative velocity of a particle in an alpha field, v_α.

Proposition 1. If the line element in an alpha field is defined by the nondiagonal form with the Riemannian metrics [33-36]

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \kappa(\alpha-\alpha')_x c\,dt\,dt - \kappa(\alpha-\alpha')_y c\,dt\,dy - \kappa(\alpha-\alpha')_z c\,dt\,dz + dx^2 + dy^2 + dz^2, \tag{1}$$

then the relative velocity of a particle in an alpha field, v_α, can be described as the function of the field parameters α and α'

$$v_\alpha = v - \frac{\kappa(\alpha-\alpha')c}{2}. \tag{2}$$

In the previous equation v is a particle velocity in the total vacuum (without any potential field), c is the speed of the light in a vacuum and κ is a constant determined by the equations (11) and (20).

Proof if the Proposition 1. The line element, given by (1), can be transformed into the new form

$$ds^2 = -c^2 dt^2 \left(\begin{array}{l} \alpha\alpha' + \dfrac{\kappa(\alpha-\alpha')_x c\,dx}{c^2 dt} + \dfrac{\kappa(\alpha-\alpha')_y c\,dy}{c^2 dt} \\ + \dfrac{\kappa(\alpha-\alpha')_z c\,dz}{c^2 dt} - \dfrac{dx^2}{c^2 dt^2} - \dfrac{dy^2}{c^2 dt^2} - \dfrac{dz^2}{c^2 dt^2} \end{array} \right). \tag{3}$$

Now, we introduce the following substitutions into (3):

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}, \quad \to$$
$$v_x^2 + v_y^2 + v_z^2 = v^2, \; \kappa(\alpha-\alpha')_x c v_x + \kappa(\alpha-\alpha')_y c v_y + \kappa(\alpha-\alpha')_z c v_z = \kappa(\alpha-\alpha') c v. \tag{4}$$

Applying the substitutions (4) to the relation (3) we obtain the new form of the line element as the function of the particle velocity, v, and alpha field parameters α and α'

$$ds^2 = -c^2 dt^2 \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha-\alpha')cv}{c^2} \right). \tag{5}$$

The related line element, valid in the Special Relativity, can be obtained from equation (5) by putting $\alpha = \alpha' = 1$. This is the transformation of the line element from the General Relativity to the Special Relativity

$$ds^2 = -c^2 dt^2 \left(1 - \frac{v^2}{c^2} \right). \tag{6}$$

The relations (5) and (6) confirm unification of SR and GR on the line elements level. Thus, the form invariant relation of the line element in an alpha field should have the form

$$ds^2 = -c^2 dt^2 \left(1 - \frac{v_\alpha^2}{c^2} \right). \tag{7}$$

In the equation (6) we have relative velocity v for particle motion in vacuum (without any potential field). In the equation (7) we have relative velocity v_α for particle motion in an alpha field. Using the identification between equations (5) and (7) we obtain the following relations:

$$\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha-\alpha')cv}{c^2} = 1 - \frac{v_\alpha^2}{c^2}, \quad \to \quad \frac{v_\alpha^2}{c^2} = 1 - \alpha\alpha' + \frac{v^2}{c^2} - \frac{\kappa(\alpha-\alpha')cv}{c^2}. \tag{8}$$

Now, we can employ the assumption of the particle velocity, v_α, given by (2)

$$v_\alpha = v - \frac{\kappa(\alpha-\alpha')c}{2}, \quad \to \quad \frac{v_\alpha^2}{c^2} = \frac{v^2}{c^2} - \frac{\kappa(\alpha-\alpha')cv}{c^2} + \frac{\kappa^2(\alpha-\alpha')^2}{4}. \tag{9}$$

It follows the comparison between the second relations in (8) and (9) that results by the following identity

$$1 - \alpha\alpha' + \frac{v^2}{c^2} - \frac{\kappa(\alpha-\alpha')cv}{c^2} = \frac{v^2}{c^2} - \frac{\kappa(\alpha-\alpha')cv}{c^2} + \frac{\kappa^2(\alpha-\alpha')^2}{4}, \quad \to \quad 1 - \alpha\alpha' = \frac{\kappa^2(\alpha-\alpha')^2}{4}. \tag{10}$$

The last relation in (10) can be transformed into the simplest form that gives the very important relation between the field parameters α and α':

$$1-\alpha\alpha' = \frac{\kappa^2(\alpha-\alpha')^2}{4}, \quad \kappa^2 = 1, \quad \kappa = \pm 1, \qquad \rightarrow \left(\frac{\alpha+\alpha'}{2}\right)^2 = 1. \tag{11}$$

From (11) we obtain the definition of the constant κ and direct relation between field parameters α and α'. This relation will be employed in the process of the determination of the field parameters α and α'.

Following the previous consideration, we can conclude that the equation (2) describes the relative velocity of a particle in an alpha field v_α if the conditions given by (11) are satisfied:

$$v_\alpha = v - \frac{\kappa(\alpha-\alpha')c}{2}, \quad if \quad \kappa^2 = 1, \quad \kappa = \pm 1, \qquad and \quad \left(\frac{\alpha+\alpha'}{2}\right)^2 = 1. \tag{12}$$

On that way, the proof of the proposition 1 is finished.

Proposition 2. The last relation in (12) satisfies the well-known condition [1, 2, 34-36] for the metric tensor of the line element (1)

$$\sqrt{-\det\left(g_{\mu\nu}\right)} = 1. \tag{13}$$

Proof of the Proposition 2. The general Riemannian line element [34] can be introduced by the following expression

$$ds^2 = g_{00}\left(dx^0\right)^2 + 2g_{01}dx^0dx^1 + 2g_{02}dx^0dx^2 + 2g_{03}dx^0dx^3 + g_{11}\left(dx^1\right)^2 + g_{22}\left(dx^2\right)^2 + g_{33}\left(dx^3\right)^2. \tag{14}$$

Here $g_{\mu\nu}$ are the related metric tensor components of the Riemannian manifold. Using comparison of the equations (1) and (14), we can conclude that non-null components of the metric tensor $\boldsymbol{g}$ in the line element (1) are determined by the following relations:

$$g_{00} = -\alpha\alpha', \quad g_{01} = g_{10} = b_x = \frac{-\kappa(\alpha-\alpha')_x}{2},$$
$$g_{02} = g_{20} = b_y = \frac{-\kappa(\alpha-\alpha')_y}{2}, \quad g_{11} = 1, \quad g_{22} = 1, \tag{15}$$
$$g_{03} = g_{30} = b_z = \frac{-\kappa(\alpha-\alpha')_z}{2}, \quad g_{33} = 1.$$

Following the relations (14) and (15), the general form of the line element (1) can also be presented by the new expression

$$ds^2 = -\alpha\alpha'\left(dx^0\right)^2 + 2b_x dx^0dx^1 + 2b_y dx^0dx^2 + 2b_z dx^0dx^3 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \tag{16}$$

Comparing (1) and (16) we can conclude that the related contravariant coordinates of the line elements (1) and (16) are determined by the relations:

$$dx^0 = c\,dt, \; dx^1 = dx, \; dx^2 = dy, \; dx^3 = dz. \tag{17}$$

From the relations (15) an (16) one can derive a matrix expression of the components of the general covariant metric tensor $\boldsymbol{g_{\mu\nu}}$ in an alpha field

$$\left[g_{\mu\nu}\right] = \begin{bmatrix} -\alpha\alpha' & b_x & b_y & b_z \\ b_x & 1 & 0 & 0 \\ b_y & 0 & 1 & 0 \\ b_z & 0 & 0 & 1 \end{bmatrix}. \tag{18}$$

This metric tensor is symmetric and has ten non-zero elements, as we expected that should be. The matrix expression of the metric tensor (18) is nondiagonal and belongs to the well-known Riemannian metrics [34]. Therefore, the related line element (1) is also called a nondiagonal line element. In the case of vacuum, field parameters $\alpha = \alpha' = 1$ and metric tensor (18) is transformed into the well-known metric tensor in SR

$$\left[\eta_{\mu\nu}\right] = diag\left[-1 \quad 1 \quad 1 \quad 1\right]. \tag{18a}$$

The relations (18) and (18a) confirm unification of SR and GR on the metric tensor level. The determinant and the trace of the matrix (18) are presented by the relations:

$$\det\left[g_{\mu\nu}\right] = -\left(\alpha\alpha' + b^2\right), \quad T_R\left[g_{\mu\nu}\right] = 3 - \alpha\alpha', \quad b^2 = b_x^2 + b_y^2 + b_z^2. \tag{19}$$

Now, we recall the well-known condition (13) that should be satisfied by any metric tensor [1,34, 35]. Including the determinant (19) into the condition (13) we obtain the important relation between field parameters α and α':

$$\sqrt{-\det\left[g_{\mu\nu}\right]} = \sqrt{\alpha\alpha' + b^2} = \sqrt{\alpha\alpha' + \frac{\kappa^2(\alpha-\alpha')^2}{4}} = 1, \qquad \kappa^2 = 1, \; \rightarrow \left(\frac{\alpha+\alpha'}{2}\right)^2 = 1. \tag{20}$$

On that way, the proof of the proposition 2 is finished. The condition (20) is also satisfied for $\alpha = \alpha' = 1$ that is related to the particle motion in a total vacuum (without any potential field). This case belongs to the Special Theory of Relativity.

Proposition 3. Let $d\tau$ and dt are differentials of the proper time and coordinate time of the moving particle, respectively. Further, let H is a transformation factor, as an invariant of an alpha field, and v_α is a particle velocity in that field given by (2). For that case, the transformation factor H has the following form

$$H = \frac{dt}{d\tau} = \left(1 - \frac{v_\alpha^2}{c^2}\right)^{-1/2} = \left(\alpha\alpha' - \frac{v^2}{c^2} + \frac{\kappa(\alpha-\alpha')cv}{c^2}\right)^{-1/2}. \tag{21}$$

Proof of the Proposition 3. In order to prove the relation (21) we can start with the usual definition of the differential of proper time $d\tau$

$$d\tau^2 = \frac{-ds^2}{c^2} = \frac{1}{H^2}dt^2 \;\rightarrow\; H^2 = \frac{dt^2}{d\tau^2} = -\frac{1}{ds^2}c^2dt^2. \tag{22}$$

Applying the line element (5) to the second relation in (22), we obtain the second form of the transformation factor H given by (21)

$$H=\frac{dt}{d\tau}=\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa(\alpha-\alpha')cv}{c^2}\right)^{-1/2}. \qquad (23)$$

On the other hand, employing the identification given in the first relation in (8), one obtains the first form of the transformation factor H given by (21)

$$H=\frac{dt}{d\tau}=\left(1-\frac{v_\alpha^2}{c^2}\right)^{-1/2}. \qquad (24)$$

The first relation in (8) also shows that the first and the second form of the transformation factor *H* are equal each to the other.

Following the equations (23) and (24) we can conclude that the proposition 3 is proved. Furthermore, if a particle is moving in a total vacuum (without any potential field), then we have $\alpha = \alpha' = 1$, and the relation (21) is transformed into the transformation factor γ valid in the Special Relativity

$$\alpha=\alpha'=1, \rightarrow H=\frac{dt}{d\tau}=\left(1-\frac{v^2}{c^2}\right)^{-1/2}=\gamma=\frac{dt}{dt'}, \qquad (25)$$
$$\rightarrow \quad d\tau=dt'.$$

Relations (21) and (25) confirm unification of SR and GR on the transformation factor level.

III. Solution of the Field Parameters

Proposition 4. Let m_0 is a rest mass of a particle, U is a potential energy of a particle in an alpha field, c is, as usual, the speed of the light in a vacuum and (i) is an imaginary unit. In that case the field parameters α and α' can be described as dimensionless (unitless) functions of the normalized potential energy U of a particle in an alpha field. There are four solutions for both parameters α and α' in an alpha field that can be presented by the following relations:

$$f(U)=2U/m_0c^2+\left(U/m_0c^2\right)^2, \rightarrow \alpha_1=1+i\sqrt{f(U)},$$
$$\alpha'_1=1-i\sqrt{f(U)}, \quad \alpha_2=1-i\sqrt{f(U)}, \quad \alpha'_2=1+i\sqrt{f(U)},$$
$$\alpha_3=-1+i\sqrt{f(U)}, \quad \alpha'_3=-1-i\sqrt{f(U)},$$
$$\alpha_4=-1-i\sqrt{f(U)}, \quad \alpha'_4=-1+i\sqrt{f(U)}. \qquad (26)$$

Proof of the Proposition 4. Because there are two field parameters, α and α', we have to find out two equations for solution of these parameters. At the first, the field parameters α and α' should satisfy the condition given by (20)

$$\sqrt{-det\left[g_{\mu\nu}\right]}=\sqrt{\alpha\alpha'+\frac{\kappa^2(\alpha-\alpha')^2}{4}}=1, \rightarrow \qquad (27)$$
$$\alpha\alpha'+\frac{\kappa^2(\alpha-\alpha')^2}{4}=1.$$

From the equation (11) we know that parameter $\kappa=\pm 1$. Thus, we can substitute this identity into the last relation in (27). In that case we obtain the following relation

$$\alpha\alpha'+\frac{\kappa^2(\alpha-\alpha')^2}{4}=\alpha\alpha'+\frac{(\alpha-\alpha')^2}{4}=\left(\frac{\alpha+\alpha'}{2}\right)^2=1. \qquad (28)$$

Following (27) and (28) we can conclude that the well-known condition for the metric tensors given by (13) is transformed into the useful relation between field parameters α and α' that should be satisfied:

$$\frac{\alpha+\alpha'}{2}=\pm 1, \quad \rightarrow \quad \alpha=\pm 2-\alpha'. \qquad (29)$$

Using the multiplication of the last relation in (29) by field parameter α one obtains the following quadratic equation

$$\alpha^2\mp 2\alpha+\alpha\alpha'=0. \qquad (30)$$

This is the first equation that will be employed in the process of determination of the field parameters α and α'.

The second equation should connect the field parameters α and α' with the potential energy of a particle in an alpha field. In that sense, we can employ the related covariant energy equation E_c for a particle with rest mass m_0. In order to determine the covariant energy equation E_c, we can start with the components of the covariant four-momentum vector:

$$P_\mu=g_{\mu\nu}P^\nu, \quad \mu,\nu=0,1,2,3, \rightarrow$$
$$P_0=-\alpha\alpha'P^0+b_xP^1+b_yP^2+b_zP^3, \quad P_1=b_xP^0+P^1,$$
$$P_2=b_yP^0+P^2, \quad P_3=b_zP^0+P^3,$$
$$P^0=Hm_0c, \; P^1=Hm_0v^1, \; P^2=Hm_0v^2, \; P^3=Hm_0v^3. \qquad (31)$$

Here P^ν are the components of the contravariant four-momentum vector and H is given by (21). The covariant energy equation E_c in an alpha field can be derived by using the following relations:

$$P_0=\frac{-E_c}{c}, \rightarrow E_c=-P_0c=\alpha\alpha'P^0c-b_xP^1c$$
$$-b_yP^2c-b_zP^3c. \qquad (32)$$

Now, we can substitute the contravariant momentums P^ν from (31) and parameters (b_x, b_y, b_z) from (15) into (32). As the result, we obtain the covariant energy equation E_c, valid for an alpha field:

$$E_c=Hm_0\alpha\alpha'c^2+\frac{Hm_0\kappa(\alpha-\alpha')cv}{2},$$
$$H=\left(\alpha\alpha'-\frac{v^2}{c^2}+\frac{\kappa(\alpha-\alpha')cv}{c^2}\right)^{-1/2}. \qquad (33)$$

From (33) we can see that the covariant energy equation E_c is in the linear form. The same equation has also been obtained by separately derivation of the generalized relativistic Hamiltonian, $\mathcal{H}_\alpha=E_c$, in an alpha field [37]. The related nonlinear equation of E_c can be obtained by applying of the square operation to the relation (33) (see [37]):

$$\frac{E_c^2}{c^2}-P^2=\alpha\alpha'm_0c^2, \quad P=Hm_0v,$$
$$\left(\frac{E_c}{\sqrt{\alpha\alpha'}}\right)^2\frac{1}{c^2}-\left(\frac{P}{\sqrt{\alpha\alpha'}}\right)^2=m_0c^2, \qquad (33a)$$
$$E_e=\frac{E_c}{\sqrt{\alpha\alpha'}}, \; P_e=\frac{P}{\sqrt{\alpha\alpha'}}, \rightarrow \frac{E_e^2}{c^2}-P_e^2=m_0c^2.$$

Here E_e and P_e are the extended covariant energy and extended momentum, respectively. The equations in (33a) are form-invariant to the related equations in Special Relativity. In the case of vacuum (without any potential field), field parameters $\alpha = \alpha'=1$ and the relations in (33) are transformed into the equations valid in the Special Relativity:

$$E_c = \gamma m_0 c^2, \quad H \to \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \tag{33b}$$

Relations (33), (33a) and (33b) confirm unification of SR and GR on the covariant energy equation level.

Now we assume that a particle is standing in an alpha field. For that case, a particle velocity is equal to zero ($v = 0$) and the relations in (33) are transformed into the following equations:

$$v = 0 \to E_c = H m_0 \alpha\alpha' c^2, \quad H = \frac{1}{\sqrt{\alpha\alpha'}} \to \tag{34}$$
$$E_c = m_0 c^2 \sqrt{\alpha\alpha'}.$$

On the other hand, we know that the energy of the particle with rest mass m_0 standing in a potential field ($v = 0$) is equal to the sum of the rest mass energy m_0c^2 and the related potential energy U of the particle in that field [37-39]

$$E_c = m_0 c^2 + U = m_0 c^2 \left(1 + \frac{U}{m_0 c^2}\right). \tag{35}$$

Comparing the relations (34) and (35), we can recognize the following identity:

$$\sqrt{\alpha\alpha'} = \left(1 + \frac{U}{m_0 c^2}\right) \to \alpha\alpha' = \left(1 + \frac{U}{m_0 c^2}\right)^2. \tag{36}$$

This is the second equation that will be employed in the process of determination of the field parameters α and α'. By the inclusion of the second relation in (36) into (30) we obtain the quadratic equation in the following form

$$\alpha^2 \mp 2\alpha + \left(1 + \frac{U}{m_0 c^2}\right)^2 = 0. \tag{37}$$

This quadratic equation can be split into the two related quadratic equations:

$$\alpha^2 - 2\alpha + \left(1 + \frac{U}{m_0 c^2}\right)^2 = 0, \alpha^2 + 2\alpha + \left(1 + \frac{U}{m_0 c^2}\right)^2 = 0. \tag{38}$$

The first quadratic relation in (38) gives the first two solutions of the field parameter α_1 and α_2, while the second quadratic relation in (38) gives the next two solutions of the field parameter α_3 and α_4:

$$\alpha_{1,2} = 1 \pm i \sqrt{\frac{2U}{m_0 c^2} + \left(\frac{U}{m_0 c^2}\right)^2},$$
$$\alpha_{3,4} = -1 \pm i \sqrt{\frac{2U}{m_0 c^2} + \left(\frac{U}{m_0 c^2}\right)^2}. \tag{39}$$

The related four solutions of the field parameter α' can be obtained by the substitution of the parameters α from (39) into the last relation in (29):

$$\alpha'_{1,2} = 1 \mp i \sqrt{\frac{2U}{m_0 c^2} + \left(\frac{U}{m_0 c^2}\right)^2},$$
$$\alpha'_{3,4} = -1 \mp i \sqrt{\frac{2U}{m_0 c^2} + \left(\frac{U}{m_0 c^2}\right)^2}. \tag{40}$$

Thus, the four solutions of the field parameters α and α' can be obtained by the unification of the two parameter structures given by (39) and (40):

$$f(U) = 2U / m_0c^2 + \left(U / m_0 c^2\right)^2, \to$$
$$\alpha_{1,2} = 1 \pm i\sqrt{f(U)}, \quad \alpha'_{1,2} = 1 \mp i \sqrt{f(U)}, \tag{41}$$
$$\alpha_{3,4} = -1 \pm i\sqrt{f(U)}, \quad \alpha'_{3,4} = -1 \mp i\sqrt{f(U)}.$$

Because the relations in (41) are equal to the relations in (26), we conclude that the proposition 4 is proved.

Further, it is easy to prove that all $\alpha_i\alpha'_i$ pairs from (41) satisfy the relation (36) giving an invariant $\alpha\alpha'$

$$\alpha_i \alpha'_i = \left(1 + \frac{U}{m_0 c^2}\right)^2 = \alpha\alpha', \quad i = 1,2,3.4. \tag{42}$$

For calculation some of the quantities in an alpha field we often need to know the difference of the field parameters (α-α'):

$$\alpha_1 - \alpha'_1 = 2i\sqrt{f(U)}, \quad \alpha_2 - \alpha'_2 = -2i\sqrt{f(U)},$$
$$\alpha_3 - \alpha'_3 = 2i\sqrt{f(U)}, \quad \alpha_4 - \alpha'_4 = -2i\sqrt{f(U)}, \tag{43}$$
$$(\alpha_1 - \alpha'_1) = (\alpha_3 - \alpha'_3), \quad (\alpha_2 - \alpha'_2) = (\alpha_4 - \alpha'_4).$$

The obtained relations in (41), (42) and (43) are valid generally and for their calculation we only need to know potential energy U of the particle in the related potential field.

Remarks 1. From the equations (41), (42) and (43) we can see that there are three very important properties of the solutions of the field parameters α and α': a) parameters α and α' are dimensionless (unitless) field parameters, b) there are four solutions of the field parameters α and α' that reminds us to the Dirac's theory [38], and c) the quantity $\alpha\alpha'$ is an invariant related to the four solutions of the field parameters α and α'.

IV. Solution of the Field Parameters in Unified Field

When two protons meet each other in a space-time, they experience all four of the fundamental forces of nature simultaneously. Thus, if protons are present in a unified field, then the potential energy U_u between two protons at distance r is described by the well-known relation [38-42]:

$$U_u = U_e + U_g + U_w + U_s, \quad U_i = \frac{C_i^2}{r} exp(-r / R_i), \tag{44}$$
$$i = e, g, w, s.$$

Here index i = e, g, w and s, denote potential energy in electrical, gravitational, weak and strong interactions, respectively, C_i^2 is a strength of the interaction and R_i is range of interaction in i-th field. The potential energy associated with each force acting between two protons is characterized by both the strength of the interaction and the range over which the interaction takes place. In each case the strength is determined by a coupling constant, and the range is characterized by the mass m_m of the exchanged particle. In each case the interaction is due to the exchange of some particle whose mass m_m determined the range of the interaction, $R = h/m_m c$, where h is Planck's constant and c is the speed of the light in vacuum. The exchanged particle is said to mediate the interaction.

The four solutions of the field parameters α and α' for a particle in a unified field can be obtained by the substitution of the potential energy U_u from (44) into the general relations given by (41):

$$\begin{gathered} f(U_u) = 2U_u / m_0c^2 + \left(U_u / m_0c^2\right)^2, \quad \rightarrow \\ \alpha_1 = 1 + i\sqrt{f(U_u)}, \quad \alpha'_1 = 1 - i\sqrt{f(U_u)}, \\ \alpha_2 = \alpha'_1, \quad \alpha'_2 = \alpha_1, \quad \alpha_3 = -1 + i\sqrt{f(U_u)}, \\ \alpha'_3 = -1 - i\sqrt{f(U_u)}, \quad \alpha_4 = \alpha'_3, \quad \alpha'_4 = \alpha_3, \\ U_u << m_0c^2, \left(U_u / m_0c^2\right)^2 \cong 0, \; f(U_u) = 2U_u / m_0c^2. \end{gathered} \tag{45}$$

Here (i) is an imaginary unit and m_0 is a rest mass of the particle in a unified field. The first four lines in (45) describe a strong unified field. If the quadric term is close to zero, $(U_u / m_0c^2)^2 \approx 0$, then the field parameters (45) describe a weak unified field. It is easy to prove that the all $\alpha\alpha'$ pairs from (45) satisfy the relations in (34), (35) and (36):

$$\begin{gathered} \alpha_i\alpha'_i = \left(1 + \frac{U_u}{m_0c^2}\right)^2 = \alpha\alpha', \; \sqrt{\alpha\alpha'} = \left(1 + \frac{U_u}{m_0c^2}\right), \\ v = 0, E_c = m_0c^2\sqrt{\alpha\alpha'} = m_0c^2\left(1 + \frac{U_u}{m_0c^2}\right) = m_0c^2 + U_u. \end{gathered} \tag{46}$$

Here E_c is the covariant energy of a particle standing ($v = 0$) in a unified field. The differences of the field parameters (α-α') for a particle in a unified field have the form:

$$\begin{gathered} (\alpha_1 - \alpha'_1) = (\alpha_3 - \alpha'_3) = 2\,i\sqrt{\frac{2U_u}{m_0c^2} + \left(\frac{U_u}{m_0c^2}\right)^2}, \\ (\alpha_2 - \alpha'_2) = (\alpha_4 - \alpha'_4) = -2\,i\sqrt{\frac{2U_u}{m_0c^2} + \left(\frac{U_u}{m_0c^2}\right)^2}. \end{gathered} \tag{47}$$

Remarks 2. The $\alpha\alpha'$ term is a quadratic function of the potential energy of a particle in a unified field. But the related covariant energy E_c of a particle, standing ($v = 0$) in the unified field, is a linear function of that potential energy. This transformation is obtained here on the natural way, without any a priory assumption.

V. Solution of the Field Parameters in Gravitational Field

If a particle with the rest mass m_0 is in a gravitational field, then the potential energy of the particle in that field U_g is described by the well-known relation [1-6, 39-42]:

$$\begin{gathered} U_g = \frac{C_g^2}{r} exp(-r / R_g), \; C_g^2 = -m_0GM, \; R_g = \infty, \\ \rightarrow \quad U_g = m_0 V_g = m_0A_{g0} = -\frac{m_0GM}{r}. \end{gathered} \tag{48}$$

Since in a gravitational field, the mediate particle is graviton with the mass $m_m = 0$, the range of the interaction is infinite, $R = \infty$, and the related potential energy from (44) is reduced to the relation (48). Here $V_g = A_{g0}$ is a scalar potential of the gravitational field, G is the gravitational constant, M is a gravitational mass, m_0 is a rest mass of the particle in that field and r is a gravitational radius. The four solutions of the field parameters α and α' for the particle in a gravitational field can be obtained by the substitution of the potential energy U_g from (48) into the general relations in (41):

$$\begin{gathered} i\sqrt{f(U_g)} = -\sqrt{2GM / rc^2 - \left(GM / rc^2\right)^2} = -\sqrt{\cdot}, \; \rightarrow \\ \alpha_1 = 1 - \sqrt{\cdot}, \quad \alpha'_1 = 1 + \sqrt{\cdot}, \quad \alpha_2 = \alpha'_1, \quad \alpha'_2 = \alpha_1, \\ \alpha_3 = -1 - \sqrt{\cdot}, \quad \alpha'_3 = -1 + \sqrt{\cdot}, \quad \alpha_4 = \alpha'_3, \quad \alpha'_4 = \alpha_3, \\ GM << rc^2, \rightarrow \left(GM / rc^2\right)^2 \cong 0, \rightarrow i\sqrt{f(U_g)} = -\sqrt{2GM / rc^2}. \end{gathered} \tag{49}$$

The first three lines in equations (49) describe a strong gravitational field. If the quadratic term $(GM / rc^2)^2 \approx 0$ then the field parameters (49) describe a relatively weak gravitational field as we have in our solar system. It is easy to prove that the all $\alpha\alpha'$ pairs from (49) satisfy the relations in (34), (35) and (36) for a particle that is standing ($v = 0$) in a gravitational field:

$$\begin{gathered} \sqrt{\alpha\alpha'} = \left(1 - \frac{GM}{rc^2}\right) \quad \rightarrow \quad \alpha\alpha' = \left(1 - \frac{GM}{rc^2}\right)^2, \\ v = 0 \quad \rightarrow \quad E_c = m_0c^2\sqrt{\alpha\alpha'} = m_0c^2 - \frac{m_0GM}{r}. \end{gathered} \tag{50}$$

The differences of the field parameters (α-α') for a particle in a gravitational field have the forms:

$$\begin{gathered} \alpha_1 - \alpha'_1 = -2\sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2}\right)^2}, \; \alpha_3 - \alpha'_3 = (\alpha_1 - \alpha'_1), \\ \alpha_2 - \alpha'_2 = 2\sqrt{\frac{2GM}{rc^2} - \left(\frac{GM}{rc^2}\right)^2}, \; \alpha_4 - \alpha'_4 = (\alpha_2 - \alpha'_2). \end{gathered} \tag{51}$$

Remarks 3. In the second part of RAF theory [30] it has been shown that field parameters (49) satisfy the Einstein's field equations with a cosmological constant $\Lambda = 0$. In the case of a strong static gravitational field [43-46,50], the quadratic term $(GM / rc^2)^2$ generates the related energy-momentum tensor $T_{\mu\eta}$ for the static field. For that case, we do not need to add by hand the related energy-momentum tensor $T_{\mu\eta}$ on the right side of the Einstein's field equations.

The second interpretation could be that the quadratic term $(GM / rc^2)^2$ generates the cosmological parameter Λ as a function of a gravitational radius [47] for $T_{\mu\eta} = 0$. It has been shown [48] that this solution of Λ is valid for both Planck's and cosmological scales.

In the case of a relatively weak static gravitational field, like in our solar system, the field parameters (49) satisfy the Einstein's field equations in a vacuum ($T_{\mu\eta} = 0$, $\Lambda = 0$). The general metrics of the relativistic alpha field theory [33] has been applied to the derivation of dynamic model of nanorobot motion in multipotential field [49].

VI. Solution of the Field Parameters in Unified Electrical and Gravitational Field

Let the source of the unified electrical and gravitational fields is an object with mass M, electric point charge Q and radius r. Thus, if a particle is an electron with a rest mass m_0 and an electric charge q, then the potential energy of the electron in the unified field, U_u, is described by the relation [38-42]:

$$U_e = \frac{C_e^2}{r} exp(-r/R_e),\ C_e^2 = qQ,\ R_e = \infty,$$
$$U_g = \frac{C_g^2}{r} exp(-r/R_g),\ C_g^2 = -m_0 GM,\ R_g = \infty, \tag{52}$$
$$U_u = U_e + U_g = \frac{qQ}{r} - \frac{m_0 GM}{r} = qA_{e0} + m_0 A_{g0}.$$

Since in an electrical field and in a gravitational field, the mediate particles are photon and graviton, respectively, with the null masses ($m_{ph} = 0$, $m_g = 0$), the ranges of the interactions are infinite ($R_e = \infty$, $R_g = \infty$), and the related potential energy from (44) is reduced to the relation (52). Here U_e is the potential energy of a particle in an electrical field, U_g is the potential energy of the particle in a gravitational field, A_{e0} is a scalar electric potential, A_{g0} is a scalar gravitational potential and G is a gravitational constant. The potential energy function $f(U_u)$ for this unified field can be obtained by using the following relations:

$$f(U_u) = \frac{2U_u}{m_0c^2} + \left(\frac{U_u}{m_0c^2}\right)^2,\ \frac{U_u}{m_0c^2} = \frac{qQ}{m_0rc^2} - \frac{m_0GM}{m_0rc^2},$$
$$G_e = \frac{q}{m_0} \ \rightarrow \ \frac{U_u}{m_0c^2} = \frac{G_eQ}{rc^2} - \frac{GM}{rc^2} = \frac{M_{eg}}{rc^2},$$
$$f(U_u) = \frac{2M_{eg}}{rc^2} + \left(\frac{M_{eg}}{rc^2}\right)^2. \tag{53}$$

In relation (53), parameter $G_e = q/m_0$ is the Kaluza-Klein constant [6-8], obtained here on the natural way. The four solutions of the field parameters α and α' for the particle in the unified electrical and gravitational field can be obtained by the substitution of the potential energy function $f(U_u)$ from (53) into the general relations (41):

$$f(U_u) = 2M_{eg}/rc^2 + \left(M_{eg}/rc^2\right)^2, \rightarrow \alpha_1 = 1 + i\sqrt{f(U_u)},$$
$$\alpha'_1 = 1 - i\sqrt{f(U_u)},\ \alpha_2 = \alpha'_1, \alpha'_2 = \alpha_1, \alpha_3 = -1 + i\sqrt{f(U_u)},$$
$$\alpha'_3 = -1 - i\sqrt{f(U_u)}, \alpha_4 = \alpha'_3, \alpha'_4 = \alpha_3, M_{eg} << rc^2, \rightarrow$$
$$\left(M_{eg}/rc^2\right)^2 \cong 0,\ \rightarrow\ f(U_u) = 2M_{eg}/rc^2. \tag{54}$$

Generally, relations (54) describe a strong unified field. But, if the quadratic term is close to zero, $(M_{eg}/rc^2)^2 \approx 0$, then the field parameters (54) describe a relatively weak unified field. It is easy to prove that the all $\alpha\alpha'$ pairs in (54) satisfy the relations in (34), (35) and (36):

$$\alpha_i\alpha'_i = \left(1 + \frac{M_{eg}}{rc^2}\right)^2 = \alpha\alpha',\ \sqrt{\alpha\alpha'} = \left(1 + \frac{M_{eg}}{rc^2}\right), v = 0,$$
$$\rightarrow E_c = m_0c^2\sqrt{\alpha\alpha'} = m_0c^2\left(1 + \frac{M_{eg}}{rc^2}\right) = m_0c^2 + \frac{m_0M_{eg}}{r}$$
$$= m_0c^2 + \frac{qQ}{r} - \frac{m_0GM}{r}. \tag{55}$$

Here E_c is the covariant energy of an electron standing ($v = 0$) in the unified electrical and gravitational field. The differences of the field parameters (α-α') for an electron in this unified field have the forms:

$$(\alpha_1 - \alpha'_1) = (\alpha_3 - \alpha'_3) = 2i\sqrt{\frac{2M_{eg}}{rc^2} + \left(\frac{M_{eg}}{rc^2}\right)^2},$$
$$(\alpha_2 - \alpha'_2) = (\alpha_4 - \alpha'_4) = -2i\sqrt{\frac{2M_{eg}}{rc^2} + \left(\frac{M_{eg}}{rc^2}\right)^2}. \tag{56}$$

Remarks 4. In the second part of this theory [30] it has been shown that field parameters (54) satisfy the Einstein's field equations without a cosmological constant ($\Lambda = 0$). In the case of the strong unified field, the quadratic term $(M_{eg}/rc^2)^2$ generates the related energy-momentum tensor $T_{\mu\eta}$ of the unified field. For that case, we do not need to add by hand the related energy-momentum tensor $T_{\mu\eta}$ of the unified electrical and gravitational field on the right side of the Einstein's field equations. In the case of a relatively weak unified field the quadratic term $(M_{eg}/rc^2)^2 \approx 0$, and the field parameters satisfy the Einstein's field equations in a vacuum ($T_{\mu\eta} = 0$, $\Lambda = 0$). Following this approach, the unified energy momentum tensor and geodesics equations, with the unified electrical and gravitational forces in 4D, are presented in the second [30] and third [31] parts of RAF theory, respectively.

VII. Conclusion

Recently developed Relativistic Alpha Field Theory (RAFT) has been used for the unification of Special Relativity (SR) and General Relativity (GR) into one self-consistent theory. Further, RAF theory has also been applied to the process of the unification of four fundamental interactions in the standard four dimensions (4D). The both unifications are consequences of an introduction of field parameters α and α' as the functions of the normalized potential energy of a particle in an alpha field. Thus, it has been shown that RAF theory is the adequate candidate for the mentioned unifications because it extends the applications of GR to the extremely strong gravitational field, including the Planck's scale. Since, Quantum Mechanics (QM) is also regular at the Planck's scale, the possibility of the future unification of GR and QM is also opened [32].

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