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Profile Geometric Effect of Cantilever Piezoelectric Device Using Flexural Mechanism

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***Abstract*— Vibrational energy harvesting device exploits unused resonant energies and convert them into useful electrical energy. Cantilever piezoelectric effect has been the most preferred technique and the efficiency of these devices is continuously improved. The amount of stress developed at the cantilever beam surface, on which piezoelectric material is attached or deposited is directly related to the output electrical energy. There have been attempts to increase the output power and efficiency of the harvesters by modifying the surface geometry of the beam by compromising piezo electric material occupancies. In this article, we have introduced flexural profile changes to improve the stress distribution at the beam surface without compromising the piezoelectric material occupancy. Theoretical stress calculations for the modified elliptical profiles are discussed and compared with Finite Element Modelling (FEM) study and thus effective surface area utilization is improved.**

***Index Terms*— Flexure, Finite element modelling Piezoelectric device, Resonance**

I. INTRODUCTION

Piezoelectric devices are widely used in many applications such as energy harvesting, sensing, Nanopositioning, actuation etc. Sources such as mechanical stress, strain, pressure, vibrations etc. are the primary inputs to this micro energy generating device. Numerous devices were constructed to exploit the piezoelectric properties to convert the ambient vibrations into useful electrical energy

A simple cantilever type piezoelectric device can generate certain amount of electricity using ambient vibrations. There are two major elements contributing as significant integral components namely piezoelectric material and Flexible element (cantilever beam)

The amount of electrical energy generated from the cantilever mechanism depends on many factors including geometrical shapes of the beam, proof mass, amount of stress developed in the beam, type of piezoelectric materials and so on. Eventually maximum stress developed at the beam translates into the maximum output energy. Many researchers performed experimental and simulations studies to investigate the suitable beam shapes [1-3]

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Roundy et al. [4] proposed different cantilever shapes such as rectangle, trapezoidal and triangle to enhance the energy generation output. Baker et al [5] designed and experimentally evaluated that triangular surface shape beams are promising cantilever type leading to a 30% increase in power output when compared to rectangular and trapezoidal surface shapes. In this line, many researchers concluded that triangular surface shape beams could generate higher output electricity due to uniform stress distribution property. Hence, detailed analytical and experimental studies suggest that varying the geometrical surface shapes produce more electrical output without altering the profile [6-7]

In this article, we have performed profile modification and pointed out exploiting the geometry profile by introducing elliptical flexural hinge in the rectangular beam to investigate the stress distribution

II. DESIGN EQUATIONS OF PIEZOELECTRIC CANTILEVER BEAM

Consider an elastic cantilever beam (*Length L*) under a concentrated load *F* is considered for this analysis to model the deformation of the cantilever design. A single isentropic material is considered for the analysis as actual structure consists of a piezoelectric unimorph or bimorph. A simple schematic of the system is shown in Figure 1

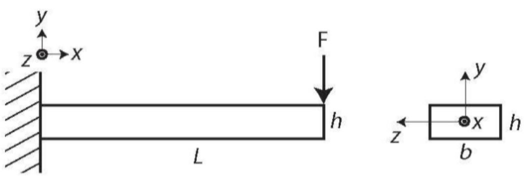

Figure 1: Cantilever beam under a concentrated load (F)

The governing differential equations for cantilever beam bending analysis are given below:

$$EI\frac{d^2u}{dx^2} = M(x) \qquad Moment \qquad (1)$$

$$\frac{du}{dx} = \theta(x) \qquad Angle\ of\ Deflection \qquad (2)$$

$$u = u(x) \qquad Deflection \qquad (3)$$

Where, E is the modulus of elasticity and I is the moment of inertia

Bending stiffness of the cantilever can be expressed as

$$k = \frac{F}{u} \tag{4}$$

Where F is the applied force and u is the deflection

The force at the end of the beam is mg. The stiffness at the end of the beam is

$$k = \frac{3EI}{L^3} \tag{5}$$

E is the modulus of elasticity. I is the area moment of inertia. L is the length

The area-moment of inertia about the z-axis is given as

$$I_z = \frac{bh^3}{12} \tag{6}$$

The natural frequency of the cantilever beam with a proof mass is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{mL^3}} \tag{7}$$

The stress at the surface of the cantilever beam can be calculated from the bending moment M_b and the sectional modulus W_y . The stress is given by

$$\sigma = \frac{M_b}{W_y} = \frac{6Fl}{bh^2} \tag{8}$$

The electric voltage generated from the deformation of the piezoelectric material is [5]

$$V = \left(\frac{3F_1 L g_{31}}{4WT}\right) \tag{9}$$

Where, g_{31} is the piezoelectric constant $\left(\frac{Vm}{N}\right)$, h is the thickness (mm), F_1 is the applied force to the piezoelectric material (N) and W is the width of piezoelectric material (mm) and T is the thickness of piezoelectric material (mm)

III. DIFFERENT CANTILEVER BEAM SURFACE SHAPES

The surface shapes of the cantilever piezoelectric generators are limited by maximum allowable stress to operate long term, fatigue becomes a concern. Electrical energy output from a piezoelectric material attached is proportional to mechanical strain applied to it. Hence, to exploit maximum potential energy in piezoelectric material, it is desirable to maximize and distribute the stress uniformly developed in cantilever beam surface.

Regrettably, conventional rectangular cantilever beam surface shape indicates that the stress concentration is accumulated at the fixture end. The maximum strain energy is transmitted to the piezoelectric material to is limited due to the lack of surface shape design modification as shown in Figure 2A. The moment induced by the inertial mass causes stress concentration at the fixture end of the beam. By providing a linearly increasing wider cross-section of the bender to support the increasing moment the beam feels, nearly uniform strain can be achieved. This suggests triangular and trapezoidal cantilever surface shapes [2] as shown in Figure 2B and 2C

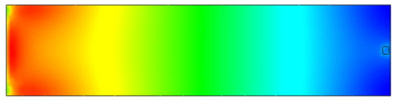

Figure 2A: Rectangular beam

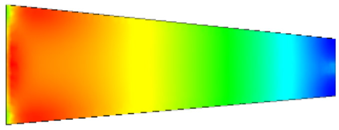

Figure 2B: Trapezoidal beam

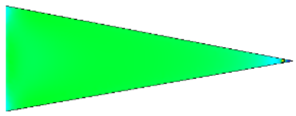

Figure2C: Triangular beam

Finite element model shows that Triangular cantilever beam is preferred over trapezoidal and rectangular because of effective surface area utilization by distributing the stress uniformly along the surface. As a result of uniform stress distribution in the beam will directly impact the electrical energy output of piezoelectric material attached to it. However, in order to have completely uniform strain, further optimization of the beam footprint would need to be performed as shown in Figure 3B and thus we introduced profile modification to investigate stress distribution

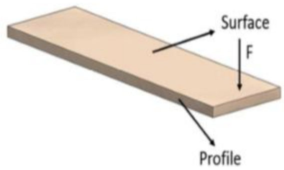

Figure: 3A

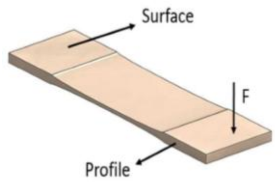

Figure: 3B

Figure 3A: Cantilever beam with uniform profile and Figure 3B: Cantilever beam with modified profile

IV. INTRODUCING PROFILE MODIFICATION IN CANTILEVER BEAM

Introducing flexure in cantilever piezoelectric device is fairly a new concept as shown in Figure 4. Bending moment is largely governed by profile thickness and has a major impact on output electrical energy. This section introduces different elliptical profile in beam to investigate the stress distribution.

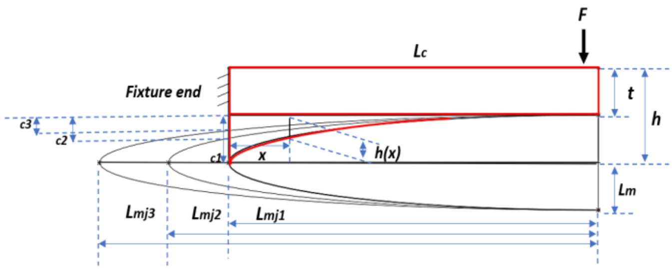

Figure 4: Parameters defining elliptical flexural hinge

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The beam compliance and stress distribution contribute output electrical energy by straining the piezoelectric material. In conventional cantilever designs, maximum stress concentration occurs at the fixed end thus the effective energy generation area is limited. In order to exploit the larger surface area, elliptical flexural hinge containing minor axis (Lm) and major axis (Lmj) is introduced in beam profile. Here origin of the major axis (Lmj1) is exactly at the fixture end of the cantilever. The varying beam thickness $h(x)$ after introducing elliptical flexure hinge can be calculated from the equation (10) (chen et al. [8]). The theoretical stress along the beam can be calculated by substituting the varying thickness in equation (8)

$$h(x) = t + c_x\left[1 - \sqrt{1 - \left(\frac{x}{L_c}\right)^2}\right] \qquad (10)$$

Parameters	Definition of the beam
Lc	Length
b	Width
$h = t+c_x$	Thickness at the fixture end
c_x	Extended thickness
t	Thickness at the proof mass end
F	Applied force
x	Any arbitrary point
Lmj1, Lmj2 andLmj3	Length of major axis
Lm	Length of the minor axis

To obtain more compliance and uniform stress distribution, the starting point of the major axis is shifted further behind to obtain Lmj2 and Lmj3(as show in figure 4) yielding different extended thickness such as c1, c2 and c3. Theoretical modelling of elliptical profiles varying the major axis is performed and compared with FEM results

V. FEM SIMULATION RESULTS

In this section, FEM modelling and simulation was performed using Solidworks 2017 and simulationXPress for different modified profiles introducing elliptical flexures and Conditions for designing 3D modelling by applying varying thickness ($h=t+c_x$) (Summarized in table 1). Beam Length (Lc) and Width (b) were kept constant as 36mm and 8.54mm respectively. The applied tip force (F) is 0.2N and alloy steel is considered for all beam models Table 1: Elliptical flexure conditions for FEM simulations

Uniform Profile Conditions: 1) Lc=36mm 2) h=1.25mm 3) b=8.54mm	**Elliptical Profile A** Conditions: 1) Lmj1=Lc=36mm 2) h=t+c1=1.25mm (t=0.5mm; c1=0.75) 3) b=8.54mm
Elliptical Profile B Conditions: 1) Lmj2=37.75mm 2) h=t+c2=1mm (t=0.5mm; c2=0.5) 3) b=8.54mm	**Elliptical Profile C** Conditions: 1) Lmj3=60.5mm 2) h=t+c3=0.65mm (t=0.5mm; c3=0.15) 3) b=8.54mm

Table 1: Elliptical flexure conditions for FEM simulations

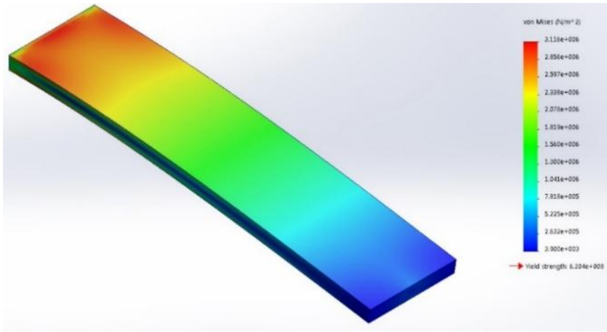

Figure 5A: Uniform profile

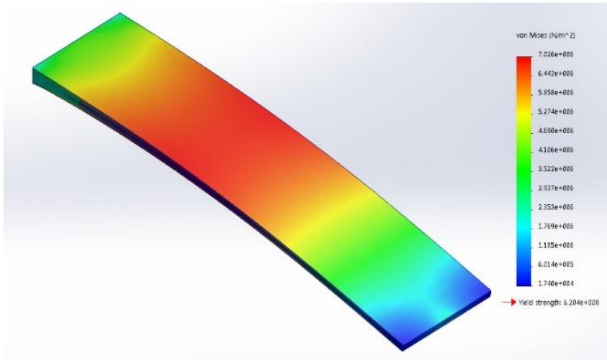

Figure 5B: Elliptical profile A

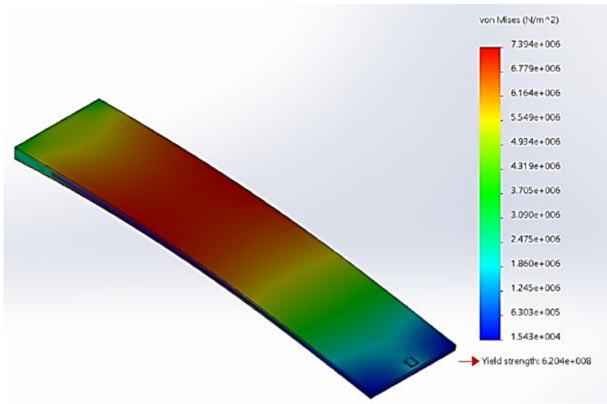

Figure 5C: Elliptical profile B

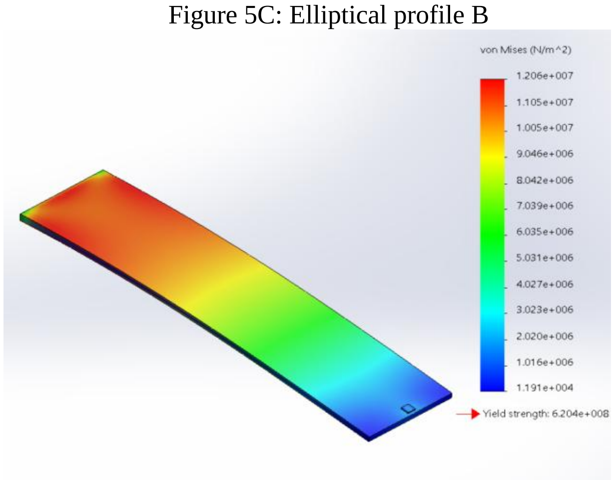

Figure 5D: Elliptical profile C

Conventional uniform cantilever without profile modification as shown in Figure 5A demonstrates that the stress concentration is highly accumulated at the fixture end and thus half of the beam goes unutilized. Piezoelectric material attached to this type of cantilever experiences higher stress loading and increases the fatigue. On the other hand, introducing an elliptical profile to the beam would significantly distribute the stress across the beam as shown in 5B and 5C. In addition, from the Figure 5C, it shows that the stress concentration (7.39N/mm^2) and distribution over the beam have been increased substantially when compared to Figure 5B. Further shifting the major axis results in increasing the beam compliance, however stress concentration is being accumulated at the fixture end as shown in Figure 5D

From the FEM study, it has been observed that major axis plays a crucial role to determine the ideal cantilever design using elliptical flexure profiles. With increasing the major axis length as shown in Figure 5D, the beam finally approaches flat spring or uniform profile and hence choosing the extended thickness (c) and Major axis are highly important. In an ideal design situation, extended thickness (c) should be equal to thickness at proof mass end (t) as shown in Figure 5C to produce uniform strain in piezoelectric material

Theoretical stress distribution was calculated using equation 6, where the varying profile thickness can be calculated using the equation 10. Figure 6 shows the comparison of theoretical and FEM stress calculations and nearly found in agreement with less than 5% variation. Elliptical profile B shows that uniform stress distribution along the beam length, hence it is preferred over other modified and conventional cantilever types shown in Figure 6. With this approach, a suitable cantilever to exploit higher energy conversion property of piezoelectric material can be designed

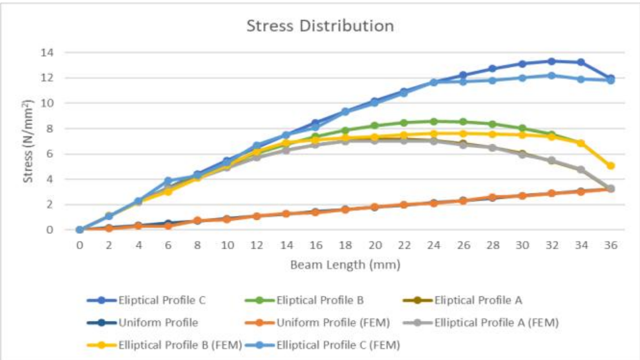

Figure 6: Theoretical and FEM beam stress

VI. Conclusion

Conventional beam surfaces such as rectangular, trapezoidal and triangular have geometric limitation to produce uniform beam stress and thus straining piezoelectric material to more electrical energy is limited. Triangular surface shape was preferred over other shapes due to its uniform stress distribution ability; however, active piezoelectric material occupancy is compromised.

Using modified profile approach, it is concluded that rectangular surface cantilever with modified elliptical profile (as show in Figure 5C: Elliptical profile B) could create more strain in piezoelectric material to produce higher output power without comprising the material occupancy

Conventional surface shapes are limited by maximum allowable stress and hence it is difficult to increase the preload/proof mass to obtain larger output electrical power. With varying profile thickness using elliptical flexural designs, it is likely to increase the preload / proof mass because of uniform stress distribution at the beam surface.
Fatigue is distributed all over the beam surface and hence, the operating life cycle of the piezoelectric device will be improved substantially

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References

[1] P. Graak et al. (2015) "Design & Simulation of Various Shapes of Cantilever Beam for Piezoelectric Power Generator" - Conference paper – COMSOL
[2] Iman.M and Francesco.B (2015) "Innovative cantilever beam shape for improved energy harvesting"- Shock & Vibration, Aircraft/Aerospace, and Energy Harvesting, Volume 9
[3] Chopra.K, Kritika.N, Sujata Pandey (2015) "Modelling and Simulation of Piezoelectric Cantilevers in RF MEMS Devices for Energy Harvesting Applications"- UKSIM-AMSS International Conference on Modelling and Simulation
[4] Roundy et al. (2005) "Improving Power Output for Vibration-Based Energy Scavengers"Published by the IEEE CS and IEEE ComSoc 1536-1268/05.
[5] Jessy Baker and Shad Roundy (2005) "Alternative Geometries for Increasing Power Density in Vibration Energy Scavenging for Wireless Sensor Networks"- Published by the American Institute of Aeronautics and Astronautics, 3rd International Energy Conversion Engineering Conference.,15 - 18 August 2005, San Francisco, California
[6] Akeel Shebeeb and Hanim Sahellah (2010) "Effect of cantilever shape on the power output of a piezoelectric bimorph generator"- ICSE2010 Proc.
[7] Marco .G and Denis.B (2013) "Finite element analysis of optimized piezoelectric bimorphs for vibrational energy harvesting"- International CAE Conference
[8] Guimin Chen et al. (2008) "A new generalized model for elliptical arc flexure hinges" - REVIEW OF SCIENTIFIC INSTRUMENTS 79, 095103.
[9] Ramadoss, T. S. and Ramakrishna, S. (2017). Human Vibration Energy Harvester with PZT. Nanotechnology for Energy Sustainability (eds B. Raj, M. Van de Voorde and Y. Mahajan). doi:10.1002/9783527696109.ch27