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Computable Criterions for an Optimal Control of Fractional Integrodifferential Systems in Banach Spaces with Distributed Delays in The Control

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***Abstract*— In this work,Fractional Integrodifferential Systems in Banach Spaces with Distributed Delays in the Control of the form:**

$$\frac{d^n x(t)}{dt^n} = Ax(t) + \int_{-h}^{0} d_\theta H(t,\theta)u(t+\theta) + f\left(t, x(t), \int_0^t g\big(t,s,x(s)\big)ds\right)$$

is presented for investigation of existence and form of an optimal control of the system .Use is made of the Unsymmetric Fubini Theorem to establish the exact mild solution of the system.The set functions upon which our results hinged are extracted from the mild solution.The concept of the game of pursuit and that of the Signum function are also used to establish results.The main result is built on the maximization of a set function,a technique drawn from the calculus of variation. Necessary and sufficient conditions for existence and form of control for the system are established.

***Index Terms*— Signum function,optimal control,mild solution,set functions, unsymmetric Fubini theorem,calculus of variation,complete state..**

I. Introduction

According to Bonilla etal (2007), fractional differential equations emerged as a new branch of mathematics. Fractional differential equations have been used for many mathematical models in Sciences and Engineering. The equations are considered as an alternative model to nonlinear differential equations. The theory of fractional differential equations has been studied extensively by many authors (.Delbosco(1996) and.Lakshimilkanthan(2008)).While the problems of stability for fractional differential systems are discussed in Bonnet (2000) , Nec and (2007),.Balachandran(2009).

Apart from stability, another important qualitative behavior of a dynamical system is controllability. Systematic study of controllability started over years at the beginning of the sixties when the theory of controllability based on the description in the form of state space for both time-varying and time-invariant linear control systems are carried out. Roughly speaking, controllability generally means that, it is possible to steer a dynamical control system from an initial state x(0) of the system to any final state x(t) in some finite time using the set of admissible controls Oraekie(2013) .The concept of controllability plays a major role in both finite and infinite dynamical systems, that is systems represented by ordinary differential equations and partial differential equations respectively. So it is natural to extend this concept to to dynamical systems represented by fractional differential equations. Many partialfractional differential equations and Integrodifferential equation can be expressed asfractional differential equations and Integrodifferential equations in Banach spaces Elsayeed (1966).

There exist many works on finite dimensional controllability of linear systems Klamka 1993)and infinite dimensional systems in abstract spacesCurtain (1978). The controllability problems of nonlinear systems and Integrodifferential systems with delays have been carried out by many researchers in both finite and infinite dimensional spaces.Balachandran (1989) and.Balachandran(2002)).Controllabilityfractional differential systems in finite dimensional space has been studied by Chen(2006) and Shamardan(2000).While Balachandran (2009) studied Controllability offractional Integrodifferential systems in Banach spaces.

In this paper, we shall consider the FractionalIntegrodifferential Systems in Banach spaces with Distributed Delays in the control of theform:

$$\frac{d^n x(t)}{dt^n} = Ax(t) + \int_{-h}^{0} d_\theta H(t,\theta)u(t+\theta) + f\left(t, x(t), \int_0^t g\big(t,s,x(s)\big)ds\right) \qquad (1.1)$$

$x(0) = x_0; t \in J = [t_0, t_1]$

with the main objective of investigating the existence and form of an optimal control the system(1.1).

II. PRELIMINARIES

Let n be a positive integer and $E = (-\infty,\infty)$ *be the real line . Denote* E^n *= the space of real n − tuples called the Euclidean space with norm denoted by* $|.|$*. If* $J = [t_0, t_1]$ *is any interval of* E*,* L_2 *is Lebesgue space of square integrable functions from* J *to* E^n *written as* $L_2([t_0,t_1], E^n)$*. Let* $h > 0$ *be positive real number and let* $C([t_0,t_1], E^n)$ *be*

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the Banach space of continuous functions with norm of uniform convergence defined by

$$\|\phi\| = sup\phi(s); \phi\epsilon\, C([t_0, t_1], E^n).$$

If x *is a function from* $[-h, \infty)$ *to* E^n, *then* x_t *is a function defined on the delay interval* $[-h, 0]$ *given as* :

$$x_t(s) = x(t-s); s \in [-h, 0], t \in [0, \infty).$$

*Definition*2.1 (*Balachandran*(2009))

The Riemann – Liouville fractional integral operator of order $\beta > 0$ *of function* $f \in C_n$, $n \geq -1$ *is defined as*:

$$I^\beta f(t) = \frac{1}{\rho(\beta)} \int_0^t (t-s)^{\beta-1} f(s) ds$$

Definition 2.2 (*fractional derivative*)

If the function $f \in C^m$ *and m is positive integer, then we can define the fractional derivative of* $f(t)$ *in the Caputo sense as*:

$$\frac{d^n f(t)}{dt^n} = \frac{1}{\rho(m-n)} \int_0^t (t-s)^{m-n-1} f^m(s) ds\, ;\ m-1 < n \leq m.$$

If $m = 1$, *then* $m-1 < n \leq m$ *becomes* $0 < n \leq 1$. *Then*

$$\frac{d^n f(t)}{dt^n} = \frac{1}{\rho(1-n)} \int_0^t (t-s)^{1-n-1} f^1(s) ds$$

$$= \frac{1}{\rho(1-n)} \int_0^t (t-s)^{-n} f^1(s) ds$$

$$= \frac{1}{\rho(1-n)} \int_0^t \frac{1}{(t-s)^n} f^1(s) ds$$

$$= \frac{1}{\rho(1-n)} \int_0^t \frac{f^1(s)}{(t-s)^n} ds,$$

where $f^1(s) = \dfrac{df(s)}{ds}$ *and f is an abstract function with values in X.*

A. VARIATION OF CONSTANT FORMULA

Consider the following system represented by the fractional Integrodifferential equations in Banach spaces with distributed delays in the control of the form:

$$\frac{d^n f(t)}{dt^n} = Ax(t) + \int_{-h}^0 d_\theta H(t,\theta)\, u(t+\theta) + f\left(t, x(t), \int_0^t g(t,s,x(s)) ds\right) \quad (1.1)$$

$$x(0) = x_0\, ; t \in J = [t_0, t_1].$$

where the state $x(.)$ *takes values in the Banach space* X, $0 < n < 1$, *the control function* $u \in L_2([t_0, t_1], U)$, *a Banach space of admissible control functions with U as a Banach space.* $H(t,\theta)$ is an nxm matrix function continuous at t and of bounded variation in θ on $[-h, 0]$, h > 0 *for each* $t \in [t_0, t_1]$; $t_1 > t_0$.

The integral is in the Lebesgue – Stieltjes sense and is denoted by the symbol d_θ. And the nonlinear operators f: JxXxX → X , g: ΔxX → X are continuous; $\Delta = \{(t,s): 0 \leq s \leq t \leq t_1\}$.

If, $\quad Gx(t) = \int_{t_0}^t g(t,s,x(s))\, ds,$

then the equation (1.1)becomes equivalent to the following nonliear integral equation

$$x(t) = x_0 + \frac{1}{\rho(n)} \int_{t_0}^t (t-s)^{n-1} Ax(s) ds + \frac{1}{\rho(n)} \int_{t_0}^t (t-s)^{n-1} \left[\int_{-h}^0 d_\theta H(t,\theta)\, u(t+\theta)\right] ds + \frac{1}{\rho(n)} \int_{t_0}^t (t-s)^{n-1} f(t, x(t), Gx(s)) ds$$

And the mild solution of the system (1.1) is given by

$$x(t) = T(t)x_0 + \frac{1}{\rho(n)} \int_{t_0}^t (t-s)^{n-1} T(t-s) \left[\int_{-h}^0 d_\theta H(t,\theta)\, u(t+\theta)\right] ds + \frac{1}{\rho(n)} \int_{t_0}^t (t-s)^{n-1} T(t-s) f(t, x(t), Gx(s)) ds \quad (1.2)$$

which is similar to the concept defined in the book of Pazy(1983).

For the limiting case, $n \to 1$, *the above system*(1.2)*reprsentation becomes*

$$x(t) = T(t)x_0 + \int_{t_0}^t T(t-s) \int_{-h}^0 d_\theta H(t,\theta)\, u(t+\theta) ds + \int_{t_0}^t T(t-s) f(t, x(t), Gx(s)) ds \quad (1.3)$$

.

Which is the mild solution of

$$\frac{dx(t)}{dt} = Ax(t) + \int_{-h}^0 d_\theta H(t,\theta)\, u(t+\theta) + f(t, x(t), Gx(s))$$

With initial condition $x(0) = x_0 \in X$.

Analogus to the conventional controllability concept. Acareful observation of the solution of the system(1.1)*given as system*(1.2)*shows that the values of the control function* $u(t)$ *for* $t \in [-h, t_1]$ *enter the definition of complete state thereby creating the need for an explicit variation of constant formula. The control in the*

$2nd\ term\ of\ the\ formula(1.2),$
$therefore, has\ to\ be\ separated\ in\ the\ intervals$ $[-h\ ,0] and\ [0\ ,t_1].$

$To\ achieve\ this\ that\ 2nd\ term\ of\ system\ (1.2) has\ to\ be\ transformed\ by\ applying\ the\ method\ of\ Klamka\ as\ contained\ in\ Chukwu(1992).$

$Finally, we\ interchange\ the\ order\ of\ integration\ using\ the\ Unsymmetric\ Fubini\ theorem\ to\ have$

$$\mathrm{x(t)} = T(t)x_0 + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)H(s,\theta)u(s+\theta)ds\right) + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} \quad (2.0)$$

$$\Rightarrow \mathrm{x(t)} = T(t)x_0 + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{t_0+\theta}^{t+\theta}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u(s-\theta+\theta)ds\right) + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} \quad (2.1).$$

$Simplifying\ system(2.1), we\ have$

$$\mathrm{x(t)} = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0(s)ds\right) + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0}^{t+\theta}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u(s)ds\right) \quad (2.2)$$

$Using\ again\ the\ Unsymmetric\ Fubuni\ Theorm\ on\ the\ change\ of\ the\ order\ of\ integration\ and\ incorporating\ H^* as\ defined\ below:$

$$H^*(s-\theta,\theta) = \begin{cases} H(s-\theta,\theta), for\ s \le t \\ 0. fors \ge t \end{cases} \quad (2.3)$$

System(.2.2) becomes

$$\mathrm{x(t)} = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0(s)ds\right) + \int_{t_0}^{t}\left[\frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)u(s)\right]ds \quad (2.4)$$

$Integration\ is\ still\ in\ the\ Lebesgue\ Stieltjes\ sense\ in\ the\ variable\ \theta\ in\ H. For\ brevity, let$

$$\alpha(t\ ,s) = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} \quad (2.5)$$

$$\beta(t\ ,s) = \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0(s)ds\right) \quad (2.6)$$

$$\mu(t\ ,s) = \frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)u(s)ds \quad (2.7)$$

Substituting equations (2.5), (2.6) and (2.7) in equation (2.4), we have a precise variation of constant formula for the system (1.1) as:

$$x(t,x_0,u) = \alpha(t\ ,s) + \beta(t\ ,s) + \int_{t_0}^{t}\mu(t\ ,s)ds \ (2.8).$$

III. MAIN RESULTS

The optimal control problem can be best understood in the context of a game of pursuit Hajek(1994), Heyman(1994). The emphasis here is the search for a control energy that can steer the state of the system of interest to the target set (which can be a moving point function or a compact set function) in a minimum time. In other words, the optimal control problem is best stated as follows:

$If\ t^* = infimum\{t: A(t\ ,t_0) \cap G(t\ ,t_0) \neq \phi\ ,\ for\ t \in$ *t0 ,t1.*

$Is\ there\ an\ admissible\ control\ u^* \in U\ such\ that\ the\ solution(state\ )x(t^*)\ of\ the\ system\ with\ this\ admissible\ control\ u^*\ be\ steered\ into\ the\ target?\ Theorem(3.1)$and Theorem(3.2) that follow answer the question.

$Theorem(3.1).$

Consider system(3.1) below as a differential game of pursuit with its basic assumptions as of the system(1.1).

$$\frac{d^n f(t)}{dt^n} = Ax(t) + \int_{-h}^{0} d_\theta H(t,\theta)\, u(t+\theta) + f\left(t, x(t), \int_0^t g\big(t,s,x(s)\big)ds\right) \quad (3.1)$$

$$x(0) = x_0 \,; t \in J = [t_0, t_1].$$

where the state $x(.)$ *takes values in the Banach space* $X, 0 < n < 1,$ *the control function* $u \in L_2([t_0\,, t_1], U)$ *, a Banach space of admissible control functions with* U *as a Banach space.* $H(t,\theta)$ is an nxm matrix function continuous at t and of bounded variation in θ on $[-h, 0]$, $h > 0$ *for each* $t \in [t_0, t_1]; t_1 > t_0$. The integral is in the Lebesgue − Stieltjes sense and is denoted by the symbol d_θ. And the nonlinear operators f: JxXxX → X , g: ΔxX → X are continuous; $\Delta = \{(t,s): 0 \le s \le t \le t_1\}$.

If, $\quad \mathrm{Gx(t)} = \int_{t_0}^{t} g\big(t,s,x(s)\big)\,\mathrm{ds},$

then the equation (3.1) becomes equivalent to the following nonliear integral equation

$$\mathrm{x(t)} = x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}\,\mathrm{Ax}(s)\mathrm{ds} + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}\left[\int_{-h}^{0} d_\theta H(t,\theta)\,u(t+\theta)\right]\mathrm{ds} + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1} f\big(t,x(t),Gx(s)\big)\mathrm{ds}$$

Suppose that the attainable set $A(t\,,t_0)$ *and the target set* $G(t\,,t_0)$ are compact set functions, respectively, then there exists an admissible control $u \in U$ such that the state $z(t)$ of the weapon for the pursuit of the target satisfies system(3.1) if and only if

$$A(t\,,t_0) \cap G(t\,,t_0) \neq \phi\,, \qquad for\ t \in [t_0\,,t_1]$$

Or if and only if the system(3.1) is relatively controllable.

PROOF.

Suppose that the state $z(t)$ *of the weapon for pursuit of target satisfies system*(3.1) *on the time interval* $[t_0\,,t_1]$ *then* $z(t) \in G(t\,,t_0)$ *for* $t \in [t_0\,,t_1]$. *We need to show that there exists a state* $x(t\,,u) \in A(t\,,t_0)$ *of the system*(3.1) *suchthat* $z(t) = x(t\,,u)$ *for some* $u \in U$ *and* $t \in [t_0\,,t_1]$.

Let $\{u^m\}$ *be an infinite sequence of points in* U. *Since* U *lies in the closed interval* $[-1\,,1]$ *, it is a constraint control set and it is also a compact set. Then the infinite sequence* $\{u^m\}$ *Has a limit* u *as* $m \to \infty$.

$$i.e.\ \lim_{m\to\infty} u^m = u$$

Now, $x(t,x_0\,,u^m) \in A(t\,,t_0)$ *and from this, we have*

$$x(t,x_0\,,u^m) = \mathrm{x(t)} = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0{}^m(s)ds\right) + \int_{t_0}^{t}\left[\frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)u^m(s)\right]ds \quad (3.2)$$

Taking limit on both sides of system(3.2) *as* $m \to \infty$*, we have*

.

$$\lim_{m\to\infty} x(t,x_0\,,u^m) = \mathrm{x(t)} = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds} + \int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)\lim_{m\to\infty} u_0{}^m(s)\,ds\right) + \int_{t_0}^{t}\left[\frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)\lim_{m\to\infty} u^m(s)\right]ds \quad (3.3)$$

Implies that

$$x(t,x_0\,,u) = \mathrm{x(t)} = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-s)f\big(t,x(t),Gx(s)\big)\mathrm{ds}$$

$$+\int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0(s)ds\right)$$

$$+\int_{t_0}^{t}\left[\frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)u(s)\right]ds$$

$= x(t,x_0,u)\in A(t,t_0)$, *since* $A(t,t_0)$ *is compact.*

Thus there exists a control energy $u\in U$ *such that* $x(t,x_0,u) =$ z(t) *for* $t > t_0$.

Since z(t) $\in$ G$(t,t_0)$and also belongs to A(t,t_0), it follows that :
$A(t,t_0)\cap G(t,t_0)\neq\phi$, *for* $t\in[t_0,t_1]$.

Observation

We observed that in any game of pursuit described by a fractional Integrodifferential system in a Banach space with distributed delays in the control, it is always possible to obtain a control energy function to steer the systems state to the target in finite time. However, the next theorem (Theorem (3.2)) is, therefore, a consequence of this understanding and provides sufficient conditions for the existence of the control energy function that is capable of steering the state of the system (3.1) to the target set in minimum time.

Theorem 3.2. (Sufficient condition)

Consider system (3.1) given as :

$$\frac{d^n f(t)}{dt^n} = Ax(t) + \int_{-h}^{0} d_\theta H(t,\theta)\,u(t+\theta) + f\left(t, x(t), \int_0^t g(t,s,x(s))ds\right) \quad (3.1)$$

$$x(0) = x_0\,;\, t\in J = [t_0,t_1].$$

With its basic assumptions.

Suppose that the system(3.1)*is relatively controllable on the finite interval* $J = [t_0,t_1]$,

Then there exists an optimal control. Onwuatu (1993) .

PROOF

By the definition of relative controllability, the intersection condition holds

i.e.$A(t,t_0)\cap G(t,t_0)\neq\phi$, *for* $t\in[t_0,t_1]$

then there exists a solution $z(t)\in A(t,t_0)$ *such that* $z(t)\in G(t,t_0)$, *for* $z(t)$*arbitrary.*

This implies that $z(t) = x(t,x_0,u)$.

Hence , $x(t,x_0,u)\in A(t,t_0)$ *and* $x(t,x_0,u)\in G(t,t_0)$

Recall that an attainable set $A(t,t_0)$ *is a translation of reachable set through the origin* η

i.e. $A(t,t_0) = \eta + R(t,t_0)$.

Here, $\eta = T(t)x_0 + \frac{1}{\rho(n)}\int_{t_0}^{t}(t-s)^{n-1}T(t-$ *sft,xt,Gxs*ds

$$+\int_{-h}^{0} d_{H_\theta}\left(\frac{1}{\rho(n)}\int_{0+\theta}^{0}(t-s)^{n-1}T(t-s)H(s-\theta,\theta)u_0(s)ds\right).$$

Thus, it follows that $z(t)\in R(t,t_0)$ *for* $t\in[t_0,t_1]$, $t > t_0$ *can be defined as* :

$$Z(t) = \int_{t_0}^{t}\left[\frac{1}{\rho(n)}\int_{-h}^{0}(t-s)^{n-1}T(t-s)d_\theta H^*(s-\theta,\theta)u(s)\right]ds.$$

Now, let $t^* = infimum\{t : z(t)\in A(t,t_0), t\in[t_0,t_1]\}$ *and* $0\le t_m\le t_1$

which implies that $\{t_m\}$ *is an infinite sequence of times and a corresponding infinite sequence of controls* $\{u_m\}\subset U$, *with* $\{t_m\}$ *converging to* t^* *− the minimum time and* $\{u_m\}$ *converging to* u^* *− the optimal control*

Let $z(t_m) = y(t_m,u_m)\in R(t,t_0)$, *then*

$$|z(t^*) - y(t^*,u_m)| = |z(t^*) - z(t_m) + z(t_m) - y(t^*,u_m)|$$
$$\le |z(t^*) - z(t_m)| + |z(t_m) - y(t^*,u_m)|$$
$$\le |z(t^*) - z(t_m)| + |y(t_m,u_m) - y(t^*,u_m)|$$
$$\le |z(t^*) - z(t_m)| + \int_{t^*}^{t_m}\|y(s,u_m)\|\,ds.$$

By the continuity of $z(t)$*which follows the continuity of* $R(t,t_0)$*as a continuous set function and the integrability of* $\|y(t)\|$, *it follows that*

$y(t^*,u_m)\to$ z(t*)

as $m\to\infty$, *where* $z(t^*) = y(t^*,u^*)\in R(t,t_0)$.

For some $u^*\in U$ *and by definition of* t^*,

u^* *is an optimal control. This establishes the existence of an optimal control for the fractional integrodifferential system*(3.1) *vis − a − vis system*(1.1).

IV. CONCLUSION

The explicit variation of constant formula for the system (1.1) visa-à-via system (3.1) was established using the Unsymmetric Fubini theorem. The set functions upon which our studies hinged were extracted from the Mild Solution.

According to Hajek (1994), Hayman (1994),the optimal control problem can be best understood in the context of a game of pursuit. Here, the emphasis is the search for a control energy function that can steer the state of the system of interest to the target set in a minimum time.

Employing this context of a game of pursuit, we stated and proved in theorem(3.1), that in any game of pursuit described by a Fractional Integrodifferential Systems in Banach Spaces with Distributed Delays in the Control, it is always possible to obtain a control energy function to steer the systems' state to the target set in finite time.

While the sufficient conditions for the existence of the control energy function (optimal control) which is capable of steering the state of system (3.1) visa-a--via system (1.1) to the target set in minimum time is for the system (3.1) visa-a--via system (1.1) to be relatively controllable. This sufficient condition was stated and proved in theorem (3.2).

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