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Relativistic Modification of Newtonian Gravity Law

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***Abstract*— We show how Newtonian gravity with effective (actually observed) masses, obeying the mass-energy relation of special relativity, can explain all observations used to test General relativity. Dynamics of a gravitationally coupled binary system is considered in detail and the effective masses of constituents are determined. Interpreting our results in terms of motion in a curved space-time background, we are led, using the Lagrangian formalism, to consider the exponential metric as a natural space-time metric of Newtonian gravity.**

***Index Terms*— Gravity . dynamics, relativity.**

I. Introduction

Newtonian dynamics introduced into scientific terminology the concepts of inertial and the gravitational mass, which for Newton were mutually proportional quantities.The exact equality [1] between the inertial and gravitational mass was postulated by Einstein [2] as the Principle of Equivalence upon which the General Theory of Relativity was founded. Modern physical theories widely use two other concepts of mass, the invariant bare or proper mass [3] and the observer depended effective mass and distance [4]. The effective mass can be viewed as a dressed bare mass due to its interaction with the surrounding medium, the space-time background for example. In this paper, we reconsider the motion of a point-like objects through a gravitational background. The interaction with the e.g. background is described by the Newtonian-like gravity force with effective (actually observed) masses and distance, obeying the Einsteins mass-energy relation, $E = mc^2$. The connection between the bare and effective mass is then given by $m_0 gdtg/dt$,where t and t g are the observer time and the gravitational time, respectively, while denotes the proper time of the moving object. The gravitational time is the time which shows the clock locally attached to the gravitational field.

II. Gravitational two body problem

In classical Newtonian mechanics the gravitational two body problem with inertial (bare) masses is exactly solvable in an analytical form. However, these solutions fail to explain the observed facts such as the motion of the planetary perihelion, the starlight deflection around the Sun, and the gravitational red shift. We shall show now that Newtonian gravity with effective masses, obeying the mass-energy relation $E = mc^2$ can give the satisfactory explanations to all the observations used to test General relativity, without invoking the field equations of General relativity.

Let the modificated Newtonian law of gravity so that the masses of two bodies and spacing points determine the material (measured, ordered) observer:

$$\vec{F}_{12} = -G\frac{m_1 m_2}{|\vec{r}_{12}|^3}|\vec{r}_{12}| \quad (1)$$

$$m_i = \frac{m_{i0}}{\sqrt{1-\dfrac{\vec{d}r_i^2}{c^2 dT^2}}}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{r}_i| = \int_0 d|\vec{r}_i|$$

$$|\vec{r}_{12}|^2 = |\vec{r}_1|^2 + |\vec{r}_2|^2 - 2|\vec{r}_1||\vec{r}_2| cos\angle(\vec{r}_1\vec{r}_2)$$

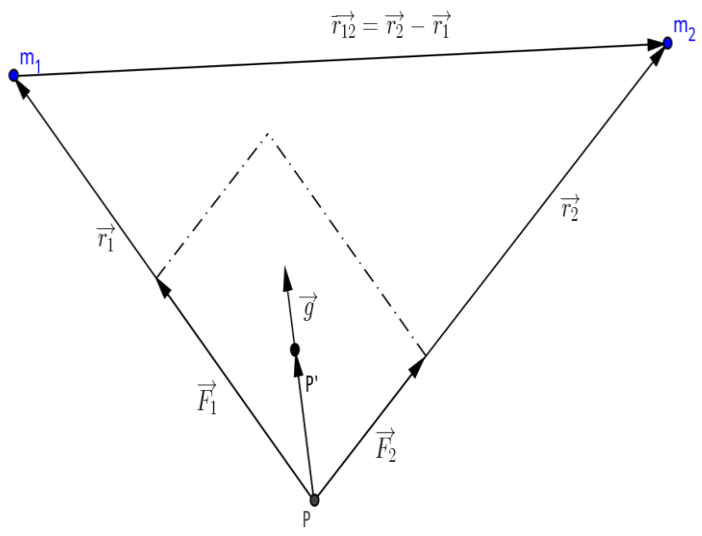

Figure 1: P' is a point in Einstein's elevator

We can consider the problem from two stand points. The first system is fixed permanently in the point P, and the

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second in the point P' which fall freely relative to P. Let: $m_1 >> m_2$ Then $(\frac{d\vec{r}_1}{dT})^2 << (\frac{d\vec{r}_2}{dT})^2$ that mass m_1 we can approximate the mass m_{10} .Select a starting point u m_1, then $\vec{r}_{12} = \vec{r}_2 = \vec{r}$

We introduce the gravitational radius $R = G\frac{m_{10}}{c^2}$ i $m_2 \equiv m$

Then force becomes:

$$\vec{F} = -\frac{Rc^2 m\vec{r}}{|\vec{r}|^3} \quad (2)$$

$$\vec{r} = \vec{r}_0|\vec{r}|$$

From these conditions the field is statical, centralsimetric and isotropic.

Let us define the environment U_m mass m as an open set containing the point m. At least one environment U_m is homeomorphic with $E^{n=4}$ (Pseudo-Minkowski space),which means that the point $m \equiv A$ can install a rectangular Cartesian system $x^k(x^0 = cT, x^{\alpha=1,2,3})$ in which each vector U_m can be represented in the following form:

$$d\vec{s} = \vec{i}_k dx^k; k = (0,1,2,3) \quad (3)$$

$\vec{i}_k$ -ortonormical base in E^4 reachable

$$\vec{i}_k\vec{i}_l = \mu_{kl} = \begin{Bmatrix} 1 & k = l = 1,2,3 \\ -1 & k = l = 0 \\ 0 & k \neq l \end{Bmatrix}$$

$$\mu_{ij}dx^i dx^j = (dx^\alpha)^2 - (dx^0)^2 = -ds^2$$

we use Einstein's convention (add on index which repeat)

Momentum $\vec{P} = m\frac{d\vec{r}}{dT} = \vec{i}_\alpha m\frac{dx^\alpha}{dT} = \vec{i}_\alpha P^\alpha$;

$$m = \frac{m_0}{\sqrt{1 - \frac{d\vec{r}^2}{c^2 dT^2}}} \Rightarrow$$

$$\vec{P}^2 - m^2c^2 = -m_0^2c^2 = -P^2 = \mu_{ij}P^iP^j; P^0 = mc \quad ;$$

$$P = (\vec{P}, P^0)$$

P represents the four-momentum with the time component

$$P^0 = mc = \frac{E}{c}$$

In E^4 we assume Newton's second law of mechanics to be valid:

$$\vec{F} = \frac{d\vec{P}}{dT} = \frac{dm}{dT}\frac{d\vec{r}}{dT} + m\frac{d^2\vec{r}}{dT^2} \quad (4)$$

Let us define the equations of motion 4) for our two observers in the point A and A' respectively.Force 2) on the entire U_m is constant, namely:

$$\vec{F}(B) = \vec{F}(A) + (\frac{\partial\vec{F}}{\partial\vec{r}})_A d\vec{r} + \frac{1}{2!}(\frac{\partial^2\vec{F}}{\partial\vec{r}}\vec{r})_A d\vec{r}^2 + ... = \vec{F}(A)$$

; $\forall B \varepsilon U_m$.

From:

$$\vec{F}d\vec{r} = \vec{F}(A)d\vec{r} = dW = -dU = dE_k = c^2 dm = -\frac{Rmc^2 d|\vec{r}|}{|\vec{r}|^2 dT}$$

where

U, E_k, m, m_T

potential energy, kinetic energy, mass and heavy mass respectively,

provided that:

$$m = \frac{m_0}{\sqrt{1 - \frac{d\vec{r}^2}{c^2 dT^2}}} = m_T$$

We take it that the inert and heavy mass are equal and come to the folowing:

$$\frac{dm}{dT} = -\frac{Rmc^2 d|\vec{r}|}{|\vec{r}|^2 dT} \quad (5)$$

$$m = m_\infty e^{\frac{R}{|\vec{r}|}}$$

$$U(\vec{r}) = -mc^2(1 - e^{-\frac{R}{|\vec{r}|}})$$

(5')

Since the force 2) is central, there exists:

$$m|\vec{r}|^2\dot{\varphi}_g = |\vec{r} \times \vec{p}| = |\vec{L}| = p_\varphi = const$$

the conservation of the angular momentum

By replacing: $|\vec{r}| = \frac{1}{\rho}$ i $\frac{d\vec{r}}{dT} = \frac{d\vec{r}}{d\varphi_g}\frac{d\varphi_g}{dT}$ we come to:

$$\ddot{\rho}+\rho = p^{-1}+R(\rho^2+\dot{\rho}^2) \quad (6)$$

$$p^{-1}=\frac{Rc^2m_0^2}{p_\varphi^2}\ ; \quad m^2=\frac{m_0^2}{1-\frac{p_\varphi^2}{m^2c^2}(\rho^2+\dot{\rho}^2)}$$

the motion equation for the observer in A.

For the observer in A' let us introduce the coordinates

$q^k=(q^0=ct,q^{\alpha=1,2,3})$

and the coordinates of both systems are a bijection:

$x^k=x^k(q^l)$

It is obvious,before explained equation the image 2

$$\vec{A'}B-\vec{A'}A=\vec{A}B=d\vec{r}=d\vec{r}'+\frac{\vec{g}}{2}dt^2=(\frac{\partial\vec{r}'}{\partial t})dt+\frac{\vec{g}}{2}dt^2=d\vec{r}^{'}=\vec{i}_\alpha dx^\alpha=\vec{i}\frac{\partial x^\alpha}{\partial q^\beta}dq^\beta$$

$$\Rightarrow \vec{A'}A=\vec{0}\Rightarrow A'=A$$

Neglecting in the equation 6) the differential of higher order, we conclude that the contribution of free fall in the measurement of displacement is negligible, that is:

$$d\vec{r}=d\vec{r}^{'}=\vec{i}_\alpha dx^\alpha=\vec{i}_\alpha\frac{\partial x_\alpha}{\partial q^\beta}dq^\beta=\vec{e}_\beta dq^\beta\Rightarrow \quad (7)$$

$$d\vec{r}^2=\vec{e}_\alpha\vec{e}_\beta dq^\alpha dq^\beta=g_{\alpha\beta}dq^\alpha dq^\beta$$

The base of the coordinate system in $A'=A$ is $\vec{e}_\beta$, and $g_{\alpha\beta}$ becames the fundamental tensor.

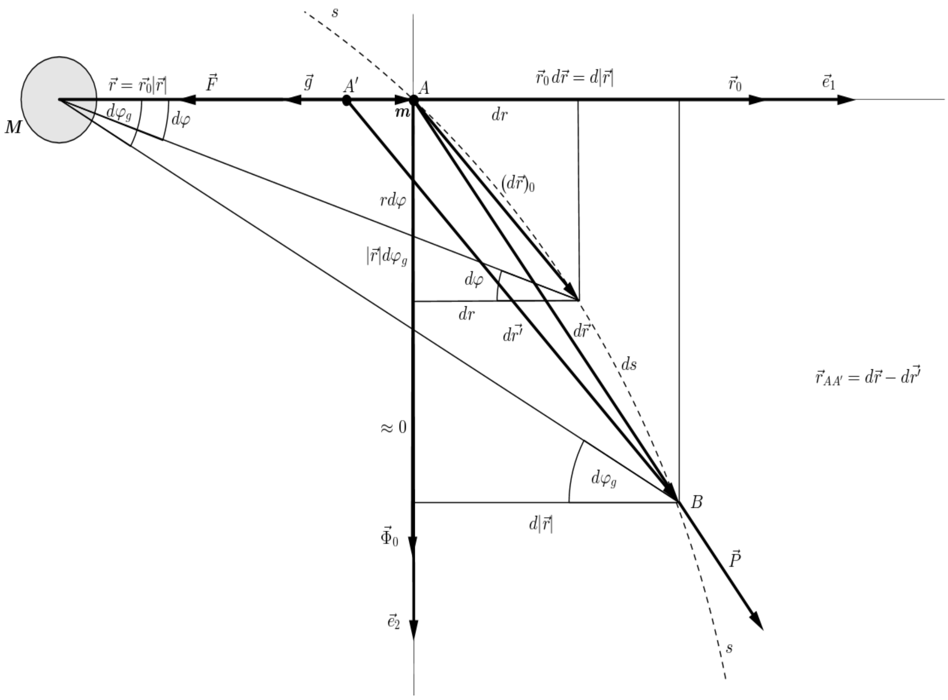

Figure 2: A'- observer in Einstein's elevator

In this case we get the following P in the base:

$$\vec{P}=\vec{e}_\alpha p^\alpha;\vec{r}=\vec{e}_\alpha t^\alpha(q^\beta);\vec{dr}=\vec{e}_\beta dq^\beta \quad (8)$$

$$\frac{\partial t^\alpha}{\partial q^\beta}+\Gamma^\alpha_{\beta_\gamma}t^\gamma=\delta_{\alpha\beta}$$

The equation of motion in this system becomes:

$$\vec{F} = \frac{d\vec{P}}{dT} / \cdot \vec{e}_\alpha$$

$$\vec{F}\vec{e}_\alpha = \vec{e}_\alpha \frac{d\vec{P}}{dT} = (\frac{dp_\alpha}{dt} - \Gamma^\nu_{\alpha\mu} p_\nu \frac{dq^\mu}{dt}) \frac{dt}{dT} = -\frac{Rc^2 mt_\alpha}{|\vec{r}|^3} \quad (9)$$

In this case the centrally symetric potential becames:

$$-dU = \vec{F}d\vec{r} = -\frac{Rc^2 mt_\alpha dq^\alpha}{|\vec{r}|^3} = \frac{\partial U}{\partial q^\alpha} dq^\alpha = \frac{\partial U}{\partial \vec{r}} d\vec{r} = \frac{\partial U}{\partial r} dr$$

$$t_2 = t_3 = 0; \vec{r} = \vec{e}_1 t^1$$

we assume that the forces in the new system are also proportional to the inverse square of distance:

$$-\frac{Rc^2 m}{r^2} = (\frac{dp_1}{dt} - \Gamma^1_{11} p_1 \frac{dr}{dt} - \Gamma^2_{12} p_2 \frac{d\varphi}{dt}) \frac{dt}{dT} \quad (10)$$

$$p_1 = \vec{P}\vec{e}_1$$

The equations of motion of the observer from A'

$$\Gamma^i_{jk} = \frac{g^{ii}}{2}(\frac{\partial q_{ij}}{\partial q^i} + \frac{\partial q_{ik}}{\partial q^j} - \frac{\partial q_{jk}}{\partial q^k})$$

By substituting: $r = \frac{1}{\rho}$ i $m = m_\infty e^{\frac{R}{r}}$; $p^{-1} = \frac{Rcm_\infty^2}{p_\varphi^2}$ 6) becomes:

$$\ddot{\rho} + \rho = p^{-1} + 4Rp^{-1}\rho + R(\rho^2 + \dot{\rho}^2) \quad (11)$$

$$\dot{\rho} = \frac{d\rho}{d\varphi}$$

We have to find determine the metric tensor of g_{ij} in the field which is static, spherosimetrical and isotropic.

g_{ij} in the statical, centralsimetric and isotropic field using the definition of Lagrangian:

$$L(q^i, q^j, t) = -m_0 c \frac{ds}{dt} = -m_0 c \sqrt{-g_{ij} \frac{dq^i}{dt} \frac{dq^j}{dt}}$$

(12)

$$g_0\alpha = (\frac{\partial x^0}{\partial q^\alpha}) = 0$$

and Legendre's transformation:

$$p_\alpha \dot{q}^\alpha - L(q^\alpha, \dot{q}^\alpha) = H(q^\alpha, p^\alpha) \quad (13)$$

From (12) and (13) to such field the following is obtained Hamiltonian:

$$H(q^\alpha, p^\alpha) = mc^2 \sqrt{-g_{00}} \quad (14)$$

from (5') follows

$$H(q^\alpha, p^\alpha) = mc^2\sqrt{-g_{00}} = \int dE = \int d(E_k + U) = \int d(mc^2 e^{\frac{-R}{r}}) = mc^2 e^{\frac{-R}{r}}$$

(15)

From 14) i 15) the following is identified:

$$-g_{00} = e^{\frac{-2R}{r}}$$

From invariant:

$$\mu_{ij} F^i dx^j = O$$

follows

$$F_1^{'} = -\frac{Rc^2 m}{|\vec{r}|^2} \frac{\partial |\vec{r}|}{\partial \vec{r}} = -\frac{Rc^2 m}{r^2}$$

From picture 2. follows

$$\sqrt{g_{11}} = \frac{\partial |\vec{r}|}{\partial \vec{r}} = \frac{|d\vec{r}|}{|d\vec{r}_0|} \Rightarrow$$

$$g_{22} = r^2 g_{11} \quad (16)$$

If the body is at rest in both systems then we get to the following conclusion:

$$dT = \sqrt{-g_{00}}\, dt$$

By analogy with the special theory of relativity:

$$|d\vec{r}| = \frac{1}{\sqrt{-g_{00}}}(|d\vec{r}|_0) = \sqrt{g_{11}}(|d\vec{r}_0|) \quad (17)$$

we get to the following equation:

$$-ds^2 = -e^{\frac{-2R}{r}} c^2 dt^2 + e^{\frac{2R}{r}} dr^2 + r^2 e^{\frac{2R}{r}} d\varphi^2 \quad (18)$$

$$\theta = \frac{\pi}{2}$$

III. Conclusion

In this paper we have analized the implications of interpreting the Newtonian gravity force in terms of actually observed effective masses obeying the mass-energy relation of special relativity. We have also stated, asserted the gravitational attraction should be accurring between all mass-equivalent energies. When using the Lagrangian formalism, the corresponding space-time background metric is found to be exponential. This metric was first introduced by Yilmaz [6] in an attempt to modify Einsteinâ€™s field equations of General relativity. However, the Yilmaz theory was immediately sharply criticized [7] on various grounds [8] as being ill defined and because it does not predict the existence of black-holes [9]. In our approach, however, the exponential metric arises as a consequence of introducing an observer in the process of measuring the gravitational attraction by means of the Newtonian gravity with effective masses. We claim that the proper (bare) mass m_0 of a body in a gravitational field will always be observed as an effective mass

$$m_g = m_o e^u$$

$$m_g = m_0$$

u . For example, on the Earths surface, the ratio mg/m0 is very close to unity, or more precisely

$$\frac{m_g}{m_0} - 1 = 7 \cdot 10^{-10}$$

The exponential metric belongs to a large family of alternative theories of gravity, which all agree with general relativity being the first order in u. In this paper, we have also demonstrated how Newtonian gravity with effective masses explains the motion of the perihelion in binary systems and the gravitational deflection of light rays. The gravitational red-shift can also be explained by our theory [10] from the relation

$$\frac{E_g(2) - E_g(2)}{E_g(1)} = e^{u(2)-u(2)} - 1 \approx u(2) - u(1) + \ldots$$

This result agrees both with the prediction of General relativity and with the observations.

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