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A New Approach To Homothetic Motions and Surfaces with Tessarines

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***Abstract*- This paper is a detailed study on homothetic motions, surfaces and Lie groups by considering the product and addition rules and conjugates of the tessarines which is given according to the arbitrary unit i_1 . To do these, we define a matrix that is similar to Hamilton operators and give some algebraic properties of this matrix. And then, we introduce different the surfaces and different hyperquadrics in R_2^4 with the help of the matrix. Also by using the homothetic motions we give some special subgroups of Lie groups and reparametrize the surfaces**

The study gives some formulas, facts and properties about homothetic motion and Lie groups that are obtained by using tessarines product and addition in R_2^4, which are not generally known.

***Index Terms*— Curves and Surfaces, Homothetic motion, Lie group, Tessarines;**

I. INTRODUCTION

First time, James Cockle defined the tessarines in 1848, using more modern notation for complex numbers as a successor to complex numbers and algebra similar to the quaternions. The tessarines are coincided with 4 -dimensional vector space R^4 over real numbers. Cockle used tessarines to isolate the hyperbolic cosine series and the hyperbolic sine series in the exponential series. He also showed how zero divisors arise in tessarines, inspiring him to use the term "impossibles." The tessarines are now best known for their subalgebra of real tessarines t=w+yj, also called split-complex numbers, which express the parametrization of the unit hyperbola , Cockle (1848) .

Homothetic motion is general form of Euclidean motion. It is crucial that homothetic motions are regular motions. These motions have been studied in kinematic and differential geometry in recent years.

In this study, we describe the tessarines and give some algebraic properties of them. Then, we show a matrix that is similar to Hamilton operators and a hyperquadric Γ in R_2^4 acquired by this matrix. We determine two different types of homothetic motions by using two different orthonormal matrices in R_2^4 and obtain the surfaces M_1 and M_2 by means of these homothetic motions.

II. PRELIMINARY

This section is devoted to some basic and fundamental concepts of tessaries. The tessarines are given by

$$W = w_0 + w_1 i + w_2 j + w_3 k$$

where the imaginary units i_1, i_2 and i_3 are governed by the rules:

$$i^2 = -1, j^2 = +1, k^2 = -1$$

and

$$ij = ji = k;\ ik = ki = -j;\ jk = kj = i.$$

Let W and U be tessarines. The addition, subtraction of these numbers are given by

$$W \mp U = (w_0 \mp u_0) + (w_1 \mp u_1) i + (w_2 \mp u_2) j + (w_3 \mp u_3) k$$

and multiplication of these numbers as follows

$$W.U = (w_0 + w_1 i + w_2 j + w_3 k).(u_0 + u_1 i + u_2 j + u_3 k) = \begin{cases} w_0u_0 - w_1u_1 - w_2u_2 + w_3u_3 \\ +i(w_0u_1 + w_1u_0 - w_2u_3 - w_3u_2) \\ +j(w_0u_2 + w_2u_0 - w_3u_1 - w_1u_3) \\ +k(w_0u_3 + w_3u_0 + w_1u_2 + w_2u_1). \end{cases}$$

It is easy to see that the multiplication of two tessarines is commutative. It is also convenient to write the set of tessarines as

$$T = \{W \mid W = w_0 + w_1 i + w_2 j + w_3 k \mid (w_{0-3}) \in R\}. \qquad (2)$$

Definition: (The conjugate of the tessarine): The conjugate of the tessarine W is shown by W^* and also there are different conjugations of tessarines according to the imaginary units i, j and k={i and k} as follows:

1. $W^* = (w_0 - w_1 i) + j(w_2 - w_3 i), = w_0 - w_1 i + w_2 j - w_3 k.$
2. $W^* = (w_0 + w_1 i) - j(w_2 + w_3 i), = w_0 + w_1 i - w_2 j - w_3 k).$
3. $W^* = (w_0 - w_1 i) - j(w_2 - w_3 i), = w_0 - w_1 i - w_2 j + w_3 k.$

The conjugation of W plays an important role both for algebraic and geometric properties for tessarines.

Multiplication of the tessarine with conjugate is given

1. $W W^* = w_0^2 + w_1^2 + w_2^2 + w_3^2 + 2j\,(w_0w_2 + w_1w_3),$
2. $W W^* = w_0^2 - w_1^2 - w_2^2 + w_3^2 + 2i\,(w_0w_1 - w_2w_3),$
3. $W W^* = w_0^2 + w_1^2 - w_2^2 - w_3^2 + 2k\,(w_0w_3 - w_1w_2).$

The system T is a commutative algebra. From equation (2), we can give the representation to show a mapping into 4x4 matrix as follows

$$\varphi : W \to \varphi(W),$$

$$\varphi(W) = \begin{bmatrix} w_0 & -w_1 & w_2 & -w_3 \\ w_1 & w_0 & w_3 & w_2 \\ w_2 & -w_3 & w_0 & -w_1 \\ w_3 & w_2 & w_1 & w_0 \end{bmatrix},$$

Yayli and Bükçü (1995), T is algebraically isomorphic to the matrix algebra

$$A = \begin{bmatrix} w_0 & -w_1 & w_2 & -w_3 \\ w_1 & w_0 & w_3 & w_2 \\ w_2 & -w_3 & w_0 & -w_1 \\ w_3 & w_2 & w_1 & w_0 \end{bmatrix}$$

and $\varphi(W)$ is a faithful real matrix representation of T. Moreover, $\forall W, U \in T$ and $\forall \lambda \in R$, we obtain

$i.\ W = U \Leftrightarrow \varphi(W) = \varphi(U)$
$ii.\ \varphi(WU) \Leftrightarrow \varphi(W).\varphi(U)$
$iii.\ \varphi(\lambda W) = \lambda.\varphi(W)\ ;\ \lambda \in IR$
$iv.\ \varphi(1) = I_4$

III. HOMOTHETIC MOTIONS AND LIE GROUPS WITH TESSARINES

Let us consider two different types of surfaces M_1 and M_2 as follows,

$$M_1 = \{W = (w_0, w_1, w_2, w_3) \mid w_0 w_2 + w_1 w_3 = 0\},$$
$$M_2 = \{W = (w_0, w_1, w_2, w_3) \mid w_0 w_1 - w_2 w_3 = 0\},$$

here by considering tessarines product and addition, if we denote

$$w_0 = \cos\rho, w_1 = \sin\rho, w_2 = 0, w_3 = 0$$

and

$$w_0 = \cosh\rho, w_1 = 0,\ w_2 = \sinh\rho, w_3 = 0,$$

respectively. The matrix forms of he surfaces M_1 and M_2 are defined by

$$H_1 = \begin{bmatrix} \cos\rho & -\sin\rho & 0 & 0 \\ \sin\rho & \cos\rho & 0 & 0 \\ 0 & 0 & \cos\rho & -\sin\rho \\ 0 & 0 & \sin\rho & \cos\rho \end{bmatrix}$$

and

$$H_1 = \begin{bmatrix} \cosh\rho & 0 & \sinh\rho & 0 \\ 0 & \cosh\rho & 0 & \sinh\rho \\ \sinh\rho & 0 & \cosh\rho & 0 \\ 0 & \sinh\rho & 0 & \cosh\rho \end{bmatrix},$$

respectively. We consider surfaces M_1 and M_2 by using the homothetic motions. Now let us defined as follow:

$$\varphi_1(\rho, s) = \mu(\rho) H_1\, v(s) + \gamma(\rho), \qquad (3)$$
$$\left\{ = \mu(\rho) \begin{bmatrix} \cos\rho & -\sin\rho & 0 & 0 \\ \sin\rho & \cos\rho & 0 & 0 \\ 0 & 0 & \cos\rho & -\sin\rho \\ 0 & 0 & \sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \\ v_4(s) \end{bmatrix} + \begin{bmatrix} \gamma_1(\rho) \\ \gamma_2(\rho) \\ \gamma_3(\rho) \\ \gamma_4(\rho) \end{bmatrix} \right\}$$

and

$$\varphi_2(\rho, s) = \mu(\rho) H_2\, v(s) + \gamma(\rho), \qquad (4)$$

$$\varphi_1(t, s) =$$

$$\left\{ \mu(\rho) \begin{bmatrix} \cosh\rho & 0 & \sinh\rho & 0 \\ 0 & \cosh\rho & 0 & \sinh\rho \\ \sinh\rho & 0 & \cosh\rho & 0 \\ 0 & \sinh\rho & 0 & \cosh\rho \end{bmatrix} \begin{bmatrix} v_1(s) \\ v_2(s) \\ v_3(s) \\ v_4(s) \end{bmatrix} + \begin{bmatrix} \gamma_1(\rho) \\ \gamma_2(\rho) \\ \gamma_3(\rho) \\ \gamma_4(\rho) \end{bmatrix} \right\}.$$

Now let $\varphi_1 : M_1 \rightarrow E_2^4$ and $\varphi_2 : M_2 \rightarrow E_2^4$ be immersion of the surfaces, respectively. Here the homothetic scale of the motion is given as $h(\rho)$.

$\gamma(\rho) = (\gamma_1(\rho), \gamma_2(\rho), \gamma_3(\rho), \gamma_4(\rho))$ is the translation vector and a profile curve is defined by $v(s) = (v_1(s), v_2(s), v_3(s), v_4(s))$.

Proposition: Let $\varphi_1 : M_1 \rightarrow E_2^4$ be an immersion of a surface M_1 in the semi-Euclidean 4-space and then from equation(3), M_1 can be rewritten by using tessarins product and addition rules as

$$\varphi(\rho, s) = \delta_1(\rho) \times v(s) + \gamma(\rho),$$

where " $\times$ " tessarine product, " $+$ " tessarine addition,

$$\begin{cases} \delta_1(\rho) = (\mu(\rho)\cos\rho, \mu(\rho)\sin\rho, 0, 0), \\ v(s) = (v_1(s), v_2(s), v_3(s), v_4(s)) \end{cases}$$

are the curves and $\gamma(\rho) = (\gamma_1(\rho), \gamma_2(\rho), \gamma_3(\rho), \gamma_4(\rho))$ is the translation vector.

Proof: We can define the curves δ_1, v and the translation vector $\gamma(t)$ by using tessarines. Then we can redefine the curves δ, v as follows:

$\delta_1(t) = \mu(\rho)\cos\rho + (h(\rho)\sin\rho)i,$
$v(s) = v_1(s) + v_2(s)i + v_3(s)j + v_4(s)k,$
$\gamma(\rho) = \gamma_1(\rho) + \gamma_2(\rho)i + \gamma_3(\rho)j + \gamma_4(\rho)k.$

Using the tessarine product and addition, the surface M_1 is written as

$\varphi(\rho, s) = \delta_1(\rho) \times v(s) + \gamma(\rho).$

Proposition: Let $\varphi_2 : M_2 \rightarrow E_2^4$ be an immersion of a surface M_2 in the semi-Euclidean 4-space and assume that M_2 is a surface in equation (4), then M_2 can be rewritten as

$$\varphi(\rho, s) = \delta_2(\rho) \times v(s) + \gamma(\rho),$$
$$\begin{cases} \delta_2(\rho) = (\mu(\rho)\cosh\rho, 0, \mu(\rho)\sinh\rho, 0), \\ v(s) = (v_1(s), v_2(s), v_3(s), v_4(s)) \end{cases}$$

are the curves.

Proof We denote the curves v, δ_2 and the translation vector $\gamma(\rho)$ by using tessarine. Then we can rewrite the curves v, δ_2 as follows:

$$\delta_2(\rho) = \mu(\rho)\cosh\rho + (\mu(\rho)\sinh\rho)j,$$

$v(s) = v_1(s) + v_2(s)i + v_3(s)j + v_4(s)k,$

Using the tessarine product and addition, the surface M_2 is obtained as

$\varphi(\rho, s) = \delta_2(\rho) \times v(s) + \gamma(\rho).$

Corollary: Consider H_1 the matrix representation of tessarine

$$\delta_1(\rho) = \mu(\rho)\cos t + (\mu(\rho)\sin\rho)i.$$

Then we get the surface M_1 defined by in equations (3) and (5) as

$$\varphi(\rho, s) = H_1 v(s) + \gamma(\rho).$$

Corollary: Consider H_2 the matrix representation of tessarine

$\delta_2(\rho) = \mu(\rho)\cosh\rho + (\mu(\rho)\sinh\rho)j.$ Then we get the surface M_2 defined by in equations (4) and (6) as

$\varphi(\rho, s) = H_2 v(s) + \gamma(\rho)$

The surfaces M_1, M_2 obtained by the parametrization from equatios (3) and (4) are rewritten as tessarines product of two curves in four dimensional semi-Euclidean space. Now we can rewrie the surfaces M_1, M_2 as tessarine product of a curve and a surface.

Corollary: Let $\varphi_1 : M_1 \rightarrow E_2^4$ be an immersion of a surface M_1 in the semi-Euclidean 4-space and M_1 defined by equation (3) is a surface. Then the surface M_1 can be rewritten by

$\varphi(t, s) = \xi_1(t) \times r(t, s) + \gamma(\rho)$

or

$\varphi(t, s) = H_1(t)\, r(t, s) + \gamma(\rho),$

where $\xi_1(\rho) = (\cot\rho, \sin\rho, 0, 0)$ is a circle, $H_1(t)$ is the matrix represantation of curve ξ_1,

$r(\rho, s) = \mu(\rho) v(s)$ is a.

Corollary: Let $\varphi_2 : M_2 \rightarrow E_2^4$ be an immersion of a surface M_2 in the semi-Euclidean 4-space and given a surface, M_2, defined by equation (4). Then the surface M_2 can be rewritten by

$\varphi(\rho, s) = \xi_2(\rho) \times r(\rho, s) + \gamma(\rho)$

or

$\varphi(\rho, s) = H_2(\rho) r(\rho, s) + \gamma(\rho),$

where $\xi_2(\rho) = (\cosh\rho, 0, \sinh\rho, 0)$ is a circle, $H_2(t)$ is the matrix represantation of curve $\xi_2, r(\rho, s) = h(\rho)\upsilon(s)$ is a surface.

IV. LIE GROUPS AND SOME SPECIAL SUBGROUPS WITH TESSARINES

In this section, Consider a hyperquadric Γ and a unit hyperquadric sphere S³, respectively as follows:

$$\left\{\begin{array}{l}\Gamma = \{X = (x_0, x_1, x_2, x_3) \neq 0 \mid \\ x_0x_1 - x_2x_3 = 0, ; \; f(X,X) \neq 0\},\end{array}\right\} \quad (5)$$

and

$$\left\{\begin{array}{l}S^3 = \{X = (x_0, x_1, x_2, x_3) \mid \\ x_0^2 - x_1^2 - x_2^2 + x_3^2 = 1\}.\end{array}\right\}$$

The set of tessarines can be given as follows:

$$\Gamma = \left\{\begin{array}{c}X = x^0 + x_1 i_1 + x_2 i_2 + x_3 i_3 \mid \\ x_0x_1 - x_2x_3 = 0\end{array}\right\}, \quad (6)$$

Corresponding to the operator Γ is represented by the following matrix for tessarines:

$$\Gamma_1 = \{A = \begin{bmatrix} x_0 & -x_1 & x_2 & -x_3 \\ x_1 & x_0 & x_3 & x_2 \\ x_2 & -x_3 & x_0 & -x_1 \\ x_3 & x_2 & x_1 & x_0 \end{bmatrix};$$
$$x_0x_1 - x_2x_3 = 0, ; \; f(X,X) \neq 0 \}.$$

Here $f(X,X)$ is pseudo Euclidean metric and it is given by

$$f(X,X) = x_0^2 - x_1^2 - x_2^2 + x_3^2$$

Remark: The norm of any element w on the hyperquadric Γ_1 is define by
$N_X = XX^* = f(X,X)$.

Theorem: In equations (3)and (4) given the set of Γ together with tessarine product is a Lie group.

Proof: Γ_1 is a differentiable manifold and at the same time a group with group operation obtained by matrix multiplication with tessaries. Let the group function be $. : \Gamma_1 x\, \Gamma_1 \to \Gamma_1$. Then $(X,Y) \to XY$ is differentiable. Therefore, $(\Gamma, .)$ can be defined a Lie group in order to made a isomorphism f. We consider the set of all unit tessarines on Γ by Γ_2. In that case Γ_2 is given by

$$\Gamma_2 = \{X \in \Gamma \; ; \; \|X\| = 1\}$$
$$= \{X \in \Gamma \; ; \; x_0^2 - x_1^2 - x_2^2 + x_3^2 = 1.$$

We show the matrix form of the group Γ_1 by Γ_3 .

$$\Gamma_3 = \{X \in \Gamma_1 ; \; \|X\| = 1\} \quad (7)$$

with the group operation of tessarine product, Γ_1 compose of a subgroup of Γ.

Lemma: In equation (7), Γ_1 is 2-dimensional Lie subgroup of Γ.

Theorem: In equation (6), consider a curve ξ which is obtained by using the homothetic motion with the homothetic function $h(t) = e^{\gamma t}$ and the profile curve
$\upsilon(t) = e^{\mu t}\,(\coth t; 0, \sinh t, 0)$,
where λ, μ are real constants. Then, in a Lie group Γ, a one-parameter subgroup are obtained by the curve ξ.

Proof: We can give the curve ξ as follows:

$$\xi(t) = \left\{\begin{bmatrix} \cosh t & 0 & \sinh t & 0 \\ 0 & \cosh t & 0 & \sinh t \\ \sinh t & 0 & \cosh t & 0 \\ 0 & \sinh t & 0 & \cosh t \end{bmatrix} \begin{bmatrix} e^{\mu t}\cos t \\ 0 \\ e^{\mu t}\sin t \\ 0 \end{bmatrix}\right\}$$

$$= e^{(\lambda+\mu)t}(\cosh 2t, 0, \sinh 2t, 0).$$

It can be obtained that

$$\xi(t_1) \times \xi(t_2) = \xi(t_1 + t_2)$$

The unit element of curve ξ is $\xi(0) = (1{,}0{,}0{,}0)$ and for all $t_1, t_2 \in R$ the invers element of $\xi^{-1}(t) = \xi(-t)$. So, ξ is a one parameter Lie subgroup of Γ.

Theorem: In equation (5), consider a curve υ which is obtained by using the homothetic motion in equation with the homothetic function $h(t) = e^{\gamma t}$ and the profile curve $\upsilon(t) = e^{\mu t}(\cos t, \sin t, 0, 0)$, where γ, β are real constants. Then the curve ξ is a one-parameter Lie subgroup in Lie group Γ.

Proof: We can write the curve ξ as follows:

$$\xi(t) = e^{\gamma t} \begin{bmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & \cos t & -\sin t \\ 0 & 0 & \sin t & \cos t \end{bmatrix} \begin{bmatrix} e^{\mu t}\cos t \\ 0 \\ e^{\mu t}\sin t \\ 0 \end{bmatrix}$$
$$= e^{(\lambda+\mu)t}(\cos 2t, \sin 2t, 0{,}0).$$

It can be showed that

$$\xi(t_1) \times \xi(t_2) = \xi(t_1 + t_2).$$

The unit element of curve ξ compose $\xi(0) = (1{,}0{,}0{,}0)$ and for all $t_1, t_2 \in R$ the invers element of $\xi^{-1}(t) = \xi(-t)$. Thus ξ compose one parameter Lie subgroup of Γ.

Theorem: In equation (3), consider a curve υ which is obtained by using the homothetic motion in equation with the homothetic function $h(t) = e^{\gamma t}$ and the profile curve $\upsilon(t) = e^{\mu t}(\cosh t, 0, \sinh t, 0)$,
where λ, μ are real constants. Then a one-parameter subgroup in a Lie group Γ are made up of a curve ξ.

Proof: We can write the curve ξ as follows:

$$\xi(t) = \left\{ e^{\gamma t} \begin{bmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & \cos t & -\sin t \\ 0 & 0 & \sin t & \cos t \end{bmatrix} \begin{bmatrix} e^{\mu t}\cosh t \\ 0 \\ e^{\mu t}\sinh t \\ 0 \end{bmatrix} \right\}$$

$$= e^{(\lambda+\mu)t}(\cos t\cosh t, \sin t\cosh t, \cos t\sinh t, \sin t\sinh t).$$

It can be showed that

$$\xi(t_1) \times \xi(t_2) = \xi(t_1 + t_2)$$

for all $t_1, t_2 \in R$. Thus ξ is one parameter Lie subgroup of Γ.

Corollary: In equation (4), consider a curve ξ which is obtained by using the homothetic motion in equation with the homothetic function $h(t) = 1$ and the profile curve $\upsilon(t) = (\cosh t, 0, \sinh t, 0)$. Then the curve ξ compose a one-parameter Lie subgroup in Lie group Γ_2.

Proof: By using homothetic motion is given by (4) for $h(t) = 1$ and the profile curve
$\upsilon(t) = (\cosh t, 0, \sinh t, 0)$,
we get
$\xi(t) = (\cosh 2t, 0, \sinh 2t, 0)$.
Since $\|\xi(t)\| = 1$, it implies that $\xi(t) \subset \Gamma_2$. So it is a one-parameter Lie subgroup of Γ_2.

Corollary: In equation (3), consider a curve ξ which is obtained by using the homothetic motion in equation with the homothetic function $h(t) = 1$ and the profile curve
$\upsilon(t) = (\cos t, \sin t, 0{,}0)$.
Then curve ξ compose a one-parameter Lie subgroup in Lie group Γ_2.

Proof: By using homothetic motion is given by (3) for $h(t) = 1$ and the profile curve $\upsilon(t) = (\cos t, \sin t, 0{,}0)$, we get $\xi(t) = (\cos 2t, \sin 2t, 0{,}0)$. $\|\xi(t)\| = 1$, it implies that $\xi(t) \subset \Gamma_2$. So it is a one-parameter Lie subgroup of Γ_2.

Corollary: A curve which is obtained by using the homothetic motion given by equation (3) with the homothetic function $h(t) = 1$ and the profile curve $\upsilon(t) = (cosht, 0, sinht, 0)$ is ξ . Then curve ξ is a one-parameter Lie subgroup in Lie group Γ_2.

Proof: For $h(t) = 1$ and the profile curve $\upsilon(t) = (cosht, 0, sinht, 0)$, we get

$$\xi(t) = (costcosht, sintcosht, costsinht, sintsinht).$$

Since $\|\xi(t)\| = 1$, it implies that $\xi(t) \subset \Gamma_2$. So it is a one-parameter Lie subgroup of Γ_2.

Theorem: In equation (3) , let M_1 be a surface which is obtained by using the homothetic motion with the homothetic function $h(t) = e^{\alpha t}$ and the profile curve $\upsilon(t) = e^{\beta s}(coshs, 0, sinhs, 0)$. Then the surface M_1 compose 2-dimensional Lie subgroup of Γ.

Proof: We obtain

$$\xi(t) = e^{at}\begin{bmatrix} cost & -sint & 0 & 0 \\ sint & cost & 0 & 0 \\ 0 & 0 & cost & -sint \\ 0 & 0 & sint & cost \end{bmatrix}\begin{bmatrix} e^{bs}cosht \\ 0 \\ e^{bs}sinht \\ 0 \end{bmatrix}$$

$$= e^{(a+b)t}(cos\,tcoshs, sintcoshs, costsinhs, sintsinhs).$$

Since

$$\xi(t_1, s_1) \times \xi(t_2, s_2) = \xi(t_1 + t_2, s_1 + s_2) \text{ for all } t_1, t_2, s_1, s_2 \in R,$$

the closure property is satisfied. The identity element of M_1 is ξ(0,0)=(1,0,0,0) and for all $t, s \in R$ and the inverse element of M_1 is $\xi^{-1}(t, s) = \xi(-t, -s)$. Then M_1 is a subgroup of Γ,On the other hand, since M_1 is a submanifold of Γ, it is a 2-dimensional Lie subgroup of Γ.

Corollary: In equation (3), let M_1 be a surface which is obtained by using the homothetic motion with the homothetic function $h(t) = 1$ and the profile curve $\upsilon(t) = (cosht, 0, sinht, 0)$. Then the surface M_1 is 2-dimensional Lie subgroup of Γ_2.

Proof: By using homothetic motion in equation (4) for $h(t) = 1$ and the profile curve υ(t)= (cosh t,0,sinh t,0) , we obtain

$$\xi(t, s) = (costcoshs, sintcoshs, costsinhs, sintsinhs).$$

Since $\|\xi(t, s)\| = 1$, it follows that $\xi(t, s) \subset \Gamma_2$. Hence the surface M_1 is a 2- dimensional Lie subgroup of Γ_2.

CONCLUSION

Study give us further Contributions to Homothetic Motions and surfaces with Tessarines to be Lie groups and one parameter Lie subgroups.

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