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Discrete-time Queue with Batch Geometric Arrivals and State Dependent Retention of Reneging Customers

Yutae Lee

***Abstract*—This paper considers a discrete-time queueing system with batch geometric arrivals and retention of reneging customers, in which retention probability depends on the number of customers in the system. The steady state probability distribution of the number of customers in the system is derived. Other performance measures such as mean number of customers in the system, blocking probability, abandon probability, and service completion probability are obtained. Numerical examples are also given.**

***Index Terms*—batch geometric arrivals, discrete-time queue, geometric service, reneging, state dependent retention**

I. Introduction

Customer reneging is a phenomenon observed commonly in queueing systems, where customers may leave the system before receiving their service due to the long waiting time. The queueing system with customer reneging was first analyzed by Palm [1]. A bibliography can be found in Gross et al. [2]. El-Sherbiny [3] considered a truncated heterogeneous two-server M/M/2/N queueing system with reneging and general balk function. Kumar and Sharma [4] considered a queueing system, where reneging customers may be retained for his future service owing to a certain customer retention strategy and analyzed an M/M/1/N queueing system with retention of reneging customers. Kumar and Sharma [5] considered a finite capacity Markovian queueing system with two heterogeneous servers, discouraged arrivals, reneging, and retention of reneging customers. Lee [6] [7] considered a discrete-time queueing system with retention of reneging customers.

This paper considers a discrete-time$\mathrm{Geo}^{\mathrm{X}}/D/1/N$ queue with retention of reneging customers, in which retention probability depends on the number of customers in the system. The steady state probability distribution of the number of customers in the system is derived. Other performance measures such as mean number of customers in the system, blocking probability of arriving customers, abandon probability of waiting customers, and service completion probability are obtained. Numerical examples are also given.

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Yutae Lee, Department of Information and Communications Engineering, Dongeui University, Busan, Republic of Korea. (e-mail: ylee@deu.ac.kr).

II. Model

In this section, we formulate the queueing model, which is based on the following assumptions:

1. We consider a discrete-time queueing system in which the time axis is divided into fixed-length contiguous intervals, referred to as slots.
2. Customers arrive according to a batch geometric process.
3. The numbers of arrivals during the consecutive slots are assumed to be independent and identically distributed random variables with distribution $\{a_k, k = 0, 1, \dots\}$.
4. The service of a customer can start only at a slot boundary.
5. The service time of customers is one slot.
6. The system has a buffer of finite capacity N.
7. Customers are served in FCFS order.
8. The reneging times follow a geometric probability distribution with parameter r.
9. During each slot, reneging customers may leave the queue without getting service with a probability depending on the number of customers in the system at the beginning of the slot. Reneging customers may leave the system with probability $s(n)$ when there are n customers in the system at the beginning of a slot.
10. During each slot, customer reneging occurs before customer arrivals.

III. Stationary Distribution

To model this system, we define the random variable N_k as the total number of customers in the system at the end of slot k. Then, the stochastic process $\{N_k, k \geq 0\}$ becomes a discrete-time Markov chain. The state space of this Markov chain is $\{0, 1, 2, \dots, N\}$.

Then, the probability $r_{i,j}$ can be obtained as: if $i \geq j$, then:

$$r_{i,j} = \sum_{k=j}^{i} \binom{i}{k} r^k (1-r)^{i-k} \binom{k}{j} s(i+1)^j (1-s(i+1))^{k-j}$$

Otherwise, $r_{i,j} = 0$. Note that $\binom{i}{k} r^k (1-r)^{i-k}$ and $\binom{k}{j} s(i+1)^j (1-s(i+1))^{k-j}$ are the probability that k customers among i waiting customers renege and the probability that j customers among k reneging customers leave the system at a slot, respectively.

The one-step transition probability matrix $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & r_{1,1} & r_{1,0} & 0 & \cdots & 0 & 0 \\ 0 & r_{2,2} & r_{2,1} & r_{2,0} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & r_{N-2,N-2} & r_{N-2,N-3} & r_{N-2,N-4} & \cdots & r_{N-2,0} & 0 \\ 0 & r_{N-1,N-1} & r_{N-1,N-2} & r_{N-1,N-3} & \cdots & r_{N-1,1} & r_{N-1,0} \end{pmatrix} \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} & a_N^+ \\ a_0 & a_1 & a_2 & a_3 & \cdots & a_{N-1} & a_N^+ \\ 0 & a_0 & a_1 & a_2 & \cdots & a_{N-2} & a_{N-1}^+ \\ 0 & 0 & a_0 & a_1 & \cdots & a_{N-3} & a_{N-2}^+ \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_1 & a_2^+ \\ 0 & 0 & 0 & 0 & \cdots & a_0 & a_1^+ \end{pmatrix} \quad (1)$$

where the probability a_k^+ is given by:

$$a_k^+ \equiv 1 - \sum_{i=0}^{k-1} a_i \quad (2)$$

Let $\boldsymbol{x} \equiv (x_0, x_1, \dots, x_N)$ be the stationary probability vector associated with the discrete-time Markov chain $\{N_k, k \geq 0\}$, where $x_i \equiv \lim_{k\to\infty} P\{N_k = i\}$. Then, the stationary probability $\boldsymbol{x}$ is obtained by solving $\boldsymbol{xP} = \boldsymbol{x}$ and $\boldsymbol{xe} = 1$, where $\boldsymbol{e}$ is the column vector with all elements 1.

The mean number of customers in the system is given by:

$$L = \sum_{i=1}^{N} i\, x_i \quad (3)$$

The blocking probability of an arriving customer is given by:

$$P_b = \sum_{i=0}^{N} \left(\sum_{j=i}^{N} x_j r_{j-1,j-i} \right) \left(\sum_{j=N-i+1}^{\infty} \frac{j-N+i}{j} \frac{\sum_{k=j}^{\infty} a_k}{\sum_{k=1}^{\infty} k a_k} \right)$$

IV. Numerical Examples

Let us illustrate the behavior of the discrete-time $\mathrm{Geo}^X/D/1/N$ queueing system with impatient. By using MATLAB, we compute some performance measures. We choose $a_0 = \frac{1}{9}$, $a_1 = \frac{2}{9}$, $a_2 = \frac{3}{9}$, $a_3 = \frac{2}{9}$, $a_4 = \frac{1}{9}$, $N = 4$, and $r = 0, 0.1, 0.2, \dots, 1$. We consider four cases: (1) $s = 0$, (2) $s = 0.4$, (3) $s = 0.2M$, and (4) $s = 1$, where M is the number of customers in the system at the beginning of a slot in steady state. The effect of parameters, retention probability s and reneging probability r, on the performance is illustrated here.

V. Conclusion

We considered a $\mathrm{Geo}^X/D/1/N$ queue with retention of reneging customers, in which retention probability depends on the number of customers in the system. Some performance measures were derived.

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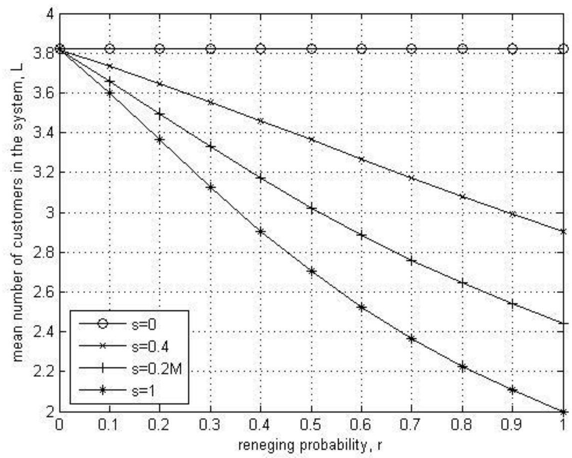

Figure 1. Mean number of customers in the system

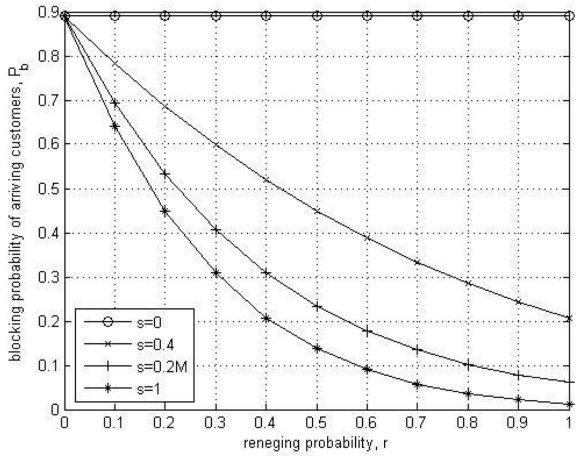

Figure 2. Blocking probability of arriving customers