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Geo/Geo/1/N Queue with State Dependent Retention of Reneging Customers

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***Abstract*—This paper considers a Geo/Geo/1/N queueing system with retention of reneging customers, in which retention probability depends on the number of customers in the system. The steady state distribution of the number of customers in the system is derived. Other performance measures are obtained. Some numerical examples are also given.**

***Index Terms*—Discrete-time queue, geometric service, reneging, state dependent retention**

I. Introduction

The phenomenon of customer reneging is commonly observed in queueing systems, where customers may leave a service system before receiving service due to the long waiting time. The queueing system with customer reneging was first analyzed by Palm [1]. A bibliography can be found in Gross et al. [2]. El-Sherbiny [3] considered a truncated heterogeneous two-server M/M/2/N queueing system with reneging and general balk function. Kumar and Sharma [4] considered a queueing system, where reneging customers may be retained for his future service owing to a certain customer retention strategy and analyzed an M/M/1/N queueing system with retention of reneging customers. Lee [5] [6] considered a discrete-time queueing system with retention of reneging customers.

This paper considers a discrete-time Geo/Geo/1/N queue with retention of reneging customers, in which retention probability depends on the number of customers in the system. The steady state distribution of the number of customers in the system is derived. Other performance measures are obtained. Some numerical examples are also given.

The paper is organized as follows. In Section II, we described the queueing model. In Section III, we formulate the system as a discrete-time Markov chain and find the stationary probability distribution of the number of customers in the system. Other performance measures are also obtained in Section III. In Section IV, some numerical examples are given. Conclusion is provided in Section V.

II. Model

We formulate the queueing model, which is based on the following assumptions:

1. We consider a discrete-time queueing system in which the time axis is divided into fixed-length contiguous intervals, referred to as slots.
2. Customers arrive according to a geometric process. Let p be the probability that a customer enters the system during a slot.
3. The service of a customer can start only at a slot boundary.
4. The service times of customers follow a geometric distribution with parameter q.
5. The system has a buffer of finite capacity N.
6. Customers are served in FCFS order.
7. The reneging times follow a geometric distribution with parameter r.
8. During each slot, reneging customers may leave the queue without getting service with a probability that depends on the number of customers in the system at the beginning of the slot. If there are n customers in the system at the beginning of a slot, then reneging customers may leave the system with probability $s(n)$.
9. During each slot, customer reneging occurs before customer arrivals.

III. Stationary Distribution

To model this system, we define a discrete-time Markov chain $\{N_k, k \geq 0\}$, where N_k denotes the total number of customers in the system at the end of slot k. The state space of this Markov chain is $\{0, 1, 2, \ldots, N\}$.

We define $r_{i,j}$ as the probability that j customers among i waiting customers leave the system due to reneging at a slot. Then, the probability $r_{i,j}$ can be obtained as: if $i \geq j$, then:

$$r_{i,j} = \sum_{k=j}^{i} \binom{i}{k} r^k (1-r)^{i-k} \binom{k}{j} s(i+1)^j (1-s(i+1))^{k-j}$$

Otherwise, $r_{i,j} = 0$. Note that $\binom{i}{k} r^k (1-r)^{i-k}$ and $\binom{k}{j} s(i+1)^j (1-s(i+1))^{k-j}$ are the probability that k customers among i waiting customers renege and the probability that j customers among k reneging customers leave the system at a slot, respectively.

The one-step transition probability matrix $\boldsymbol{P}$ is given by:

$$\boldsymbol{P} = \boldsymbol{RQ} \tag{1}$$

where:

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & r_{1,1} & r_{1,0} & 0 & \cdots & 0 \\ 0 & r_{2,2} & r_{2,1} & r_{2,0} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & r_{N-1,N-1} & r_{N-1,N-2} & r_{N-1,N-3} & \cdots & r_{N-1,0} \end{pmatrix} \tag{2}$$

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$$\boldsymbol{Q} = \begin{pmatrix} 1-p & p & 0 & \cdots & 0 & 0 \\ (1-p)q & pq+(1-p)(1-q) & p(1-q) & \cdots & 0 & 0 \\ 0 & (1-p)q & pq+(1-p)(1-q) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & pq+(1-p)(1-q) & p(1-q) \\ 0 & 0 & 0 & \cdots & (1-p)q & p+(1-p)(1-q) \end{pmatrix} \quad (3)$$

Let $\boldsymbol{x} \equiv (x_0, x_1, \dots, x_N)$ be the stationary probability vector associated with the discrete-time Markov chain $\{N_k, k \geq 0\}$, where $x_i \equiv \lim_{k\to\infty} P\{N_k = i\}$. Then, the stationary probability $\boldsymbol{x}$ is obtained by solving $\boldsymbol{xP} = \boldsymbol{x}$ and $\boldsymbol{xe} = 1$, where $\boldsymbol{e}$ is the column vector with all element 1.

The mean number of customers in the system is given by:

$$L = \sum_{i=1}^{N} i\, x_i \quad (4)$$

The blocking probability of an arriving customer is given by:

$$P_b = x_N r_{N-1,0} \quad (5)$$

The abandon probability of waiting customers is given by:

$$P_a = \frac{\sum_{i=2}^{N} x_i \sum_{j=1}^{i-1} j r_{i-1,j}}{p(1 - x_N r_{N-1,0})} \quad (6)$$

IV. NUMERICAL EXAMPLES

Let us illustrate the behavior of the discrete-time Geo/Geo/1/N queueing system with impatient customers. By using MATLAB, we compute the stationary probability distribution $\boldsymbol{x}$ for the number of customers in the system and find its mean value and the blocking probability. We choose $p = 0.5$, $q = 0.7$, $N = 4$, and $r = 0, 0.1, 0.2, \dots, 1$. We consider four cases: (1) $s = 0$, (2) $s = 0.4$, (3) $s = 0.2M$, and (4) $s = 1$, where M is the number of customers in the system at the beginning of a slot in steady state. The effect of parameters, retention probability s and reneging probability r, on the mean number of customers in the system and the blocking probability of an arriving customer is illustrated here.

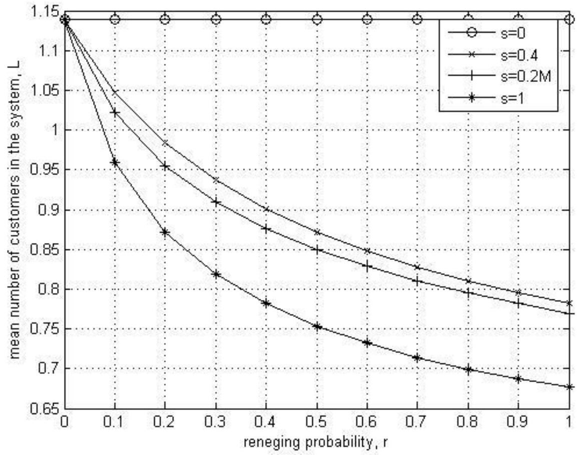

Figure 1. Mean number of customers in system

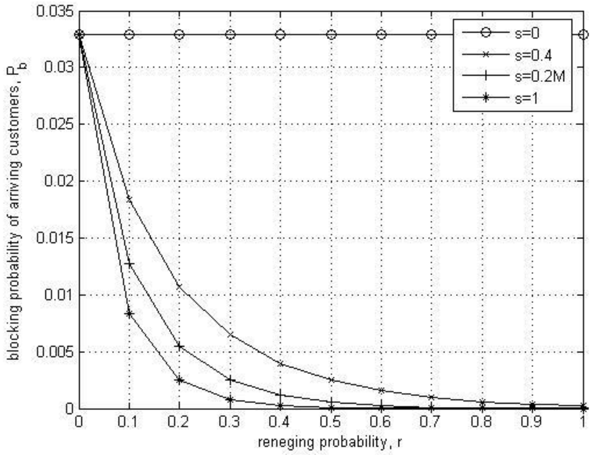

Figure 2. Blocking probability of arriving customers

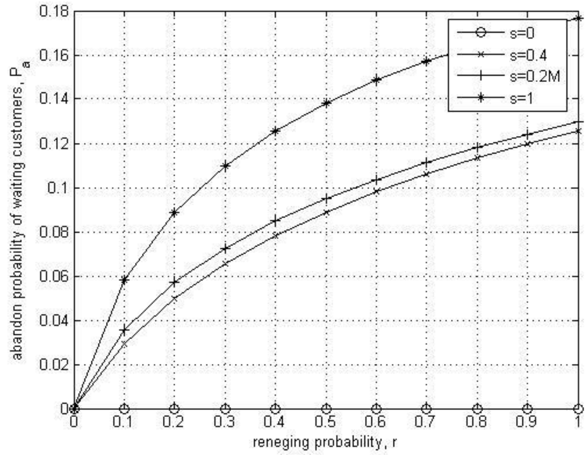

Figure 3. Abandon probability of waiting customers

V. CONCLUSION

In this paper, we have considered a Geo/Geo/1/N queue with retention of reneging customers, in which retention probability depends on the number of customers in the system. The steady state probability distribution of the number of customers in the system has been derived. Other performance measures have been obtained. Numerical examples have also been given.

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