

# Power Consumption and Equivalent Quantum State in Nano Circuits Comprising of Carbon Nano Resistors in Different Configurations

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**Abstract**— The field of carbon nano tubes (CNT) is an active and emerging area of research with many promises, theoretically as well as experimentally[1],[2]. In fact, the band structure of CNT determines its conductivity and in carbon nano tubes the structural pattern affects the conductivity type .It is well established Phenomenon that samples of single wall carbon nano tubes with the arm chair wrapping have been produced and exhibit metallic behaviour . In this paper we consider the carbon nano resistors, which are in fact conductors. To remind you all about the striking features of CNT that occur in practice such as quantized electrical behavior of resistance, resistivity, drift velocity and energy of electron in one dimensional extremely thin carbon nano tubes or wires etc, with the help of these beautiful inherent properties researchers have already advanced in many applied and theoretical aspects of CNT see for instance ref.[1],[2],[3] and [4].In the similar fashion ,we would like to concentrate on the already established quantized result of the power consumption in CNT[4] and quantized expression of current density[3] in order to determine the cumulative power consumption in electrical circuits on nano scale ,employing different and several nano resistors in simple series ,parallel and mixed grouping configurations of the resistors .Finally, some more nice results are achieved for equivalent quantum state of CNT replacing all the component quantum resistors in series, parallel and mixed grouping configurations.

**Index Terms**— Quantized, Current Density, Quantum Resistors, Power Consumption Equivalent Quantum State, Configuration.

## I. INTRODUCTION

From the point of view of the theoretical Physics ,the assumption of a single wall one dimensional carbon nano tubes(wires) having extremely small thickness, may be considered to be the fruitful starting point for further study and for the investigation of the quantized electrical behaviour of carbon nano tubes[2], [3]; even in composite structure of multiwall carbon nano tubes with several layers. But, we would not bounce back to this as,it has been already established, see for instance [1].Let us start from the quantized expression of current density of one dimensional very thin CNT, which has already been showed in research paper[3] and the quantized expression is given by

$$J = \frac{2V_{pd}}{A n \hbar} \dots\dots\dots(i)$$

$$I = \frac{2V_{pd}}{n \hbar} \dots\dots\dots(ii)$$

Where ‘A’ is the extremely small area of cross section of the nano wire, ‘ $V_{pd}$ ’ is potential difference across the carbon nano wire, ‘n’ is the quantum state of the electron in carbon nano wire or carbon quantum resistor. Respecting the quantum behavior of the conductivity, we denote ‘ $\sigma$ ’ as

$$\sigma_n = \frac{2Le^2}{A n \hbar}, \text{ therefore the resistance of the carbon nano}$$

$$\text{resistor or wire is given by } R_n = \frac{2e^2}{n \hbar} \dots\dots\dots(iii)$$

Therefore , when nano wire or carbon nano resistor is used in a simple nano circuit, then the instantaneous power consumed in the circuit can be given by,

$$\Rightarrow P_{in} = V_{pd} \cdot i \Rightarrow P_{in} = i \cdot R_n \cdot i \Rightarrow P_{in} = i^2 R_n$$

$$i^2 R_n = i^2 R_n = \left( 2V_{pd} \frac{e^2}{n \hbar} \right)^2 \times \frac{n \hbar}{2e^2} \Rightarrow i^2 R_n = 2V_{pd}^2 \frac{e^2}{n \hbar} \dots\dots\dots(iv)$$

$P_{in}$  is the instantaneous power consumed across the carbon nano resistor. Therefore, total power absorbed in time ‘t’ is total work done by the source in that given time. If we assumed the whole of work done is converted in to heat energy in SI unit we can write

$$H = W = \int_0^t P_{in} \cdot dt = \int_0^t i^2 R_n \cdot dt \dots\dots\dots(v)$$

Mean power consumed

$$\Rightarrow H = \int_0^t 2V_{pd}^2 \frac{e^2}{n \hbar} dt$$

$$\Rightarrow H = 2V_{pd}^2 \frac{e^2}{n \hbar} \int_0^t dt \dots\dots\dots(vi)$$

Since  $V_{pd}$  is independent of the time therefore , mean power consumption is equal to the instantaneous power loss

$$\bar{P} = P_i \dots\dots\dots(vii)$$

## II. METHOD AND MODEL

For dealing with our problem we consider previous model of CNT with the one dimensional approach of carbon nano tubes in which electron is confined within the well of infinite depth ,so that electron is completely free to to move with in the well .That means electron is confined from transverse directions in the tube that results in the quantized nature of the total energy of the electron[1].Hence one dimensional box model with single electron constitute the starting point for fruitful approach to find the quantized resistance and quantized power loss. Apart from this , we use well established Ohm’s law in order to find the quantized power across the resistors. the circuit in different

configuration such as series, parallel and mixed grouping etc. There is no restriction on application of Ohm's law as so far we have assumed carbon nano resistors are conductors ..Therefore , small variation of applied voltage and current would bring no appreciable change in resistance as far as validity of Ohm's law is concerned.

**THEORY:  
SERIES CONFIGURATION**

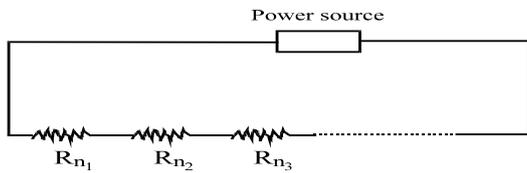


Fig-01

Let us consider a combinations of 'N' number carbon nano resistors s(carbon quantum resistor) of resistances  $R_{n1}, R_{n2}, R_{n3}, \dots$ , which are connected in series configuration. In practice, every cell has some internal resistance what so ever it smaller is .But we will ignore it for simplicity.. Since, we are dealing with conducting nano resistors in the circuit we may safely apply **Ohm's law** to quantify the current through the circui . Now, introducing the equivalent resistance in the circuit for series connection,

$$R_n = R_{n1} + R_{n2} + R_{n3} + \dots \dots \dots (v)$$

Instantaneous power consumption in the nano carbon resistor of resistance  $R_{n1}$  is ( $P_1$ ) due to flow of current, but we simply we denote it as  $P$  for convenience and same notations are being retained for other resistors.

$$P_{n1} = \frac{V_p^2}{R_{n1}} \text{ or } R_{n1} = \frac{V_p^2}{P}$$

$$R = \frac{V_p^2}{P} \text{ and so on} \dots \dots$$

Therefore the equivalent resistance in the circuit for series connection  $R_n^s = R_{n1} + R_{n2} + R_{n3} +$

$$\frac{V_p^2}{P} = \frac{V_p^2}{P_{n1}} + \frac{V_p^2}{P} + \frac{V_p^2}{P} + \dots \dots \dots$$

$$= \frac{1}{P_{n1}} + \frac{1}{P} + \frac{1}{P} + \dots \dots \dots$$

Where  $P$  is the effective power consumption of the combination of quantum resistors.

Now, switching over to the theoretical work that has already been established and cited recently that the instantaneous power consumption across a carbon quantum resistor of quantum state '  $n_1$  '  $P_{n1} = \frac{2V_p^2 \epsilon^2}{h n_1}$ , similarly for carbon

resistor of quantum state '  $n_2$  '  $P_{n2} = \frac{2V_p^2 \epsilon^2}{h n_2}$  .....

and soon. Thus, theoretically, for the sake of generalization, we consider dissimilar quantum states of all the carbon resistors,

$$R_n^s = \frac{h}{2V_p^2 \epsilon^2} (n_1 + n_2 + n_3 + \dots \dots \dots) \dots \dots \dots (viii)$$

From (vi) and (vii),we can write

=

$$\frac{h}{2V_p^2 \epsilon^2} (n_1 + n_2 + n_3 + \dots \dots \dots) \dots \dots \dots (ix)$$

Hence, we can rewrite the above two relations together as

$$P_n^s = \bar{P}_n^s \frac{2V_p^2 \epsilon^2}{h} \frac{1}{\sum_{i=1}^N n_i} \dots \dots \dots (x)$$

We have considered dissimilar quantum states of all the external carbon nano resistors .

**PARALLEL CONFIGURATION**

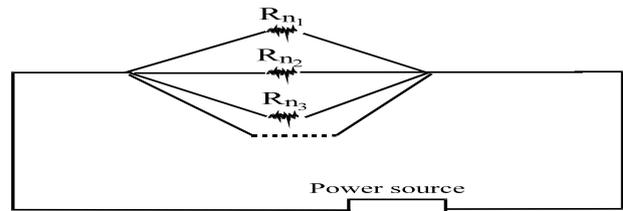


Fig-02

When all the carbon nano resistors are in parallel configuration in the circuit then we may write simply

$$\frac{1}{R_n} = \frac{1}{R_{n1}} + \frac{1}{R_{n2}} + \frac{1}{R_{n3}} + \dots \dots \dots$$

$$\text{Or } \frac{V_p^2 \epsilon^2}{R_n} = \frac{V_p^2 \epsilon^2}{R_{n1}} + \frac{V_p^2 \epsilon^2}{R_{n2}} + \frac{V_p^2 \epsilon^2}{R_{n3}} + \dots \dots \dots$$

$$\text{Or } P_n^p = P_{n1} + P_{n2} + P_{n3} + \dots \dots \dots$$

$$\text{Or } P_n^p = \frac{2V_p^2 \epsilon^2}{h} \left( \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \dots \dots \dots \right)$$

$$\text{Or } P_n^p = \frac{2V_p^2 \epsilon^2}{h} \sum_{i=1}^N \frac{1}{n_i} \dots \dots \dots (xi)$$

Note that  $\sum_{i=1}^N \frac{1}{n_i} > \frac{1}{\sum_{i=1}^N n_i}$

$$\text{Therefore, } \frac{P_n^p}{P_n^s} = \frac{1}{\sum_{i=1}^N n_i} \times \dots$$

*This is why, power loss or consumption for a given set of carbon nano resistors is more in parallel configuration compared to that of in series configuration.*

For, identical carbon nano resistors of same structural pattern and of same inherent internal property, the quantum states are expected to be identical provided they are kept in same temperature and environment in the nano circuit.

$$\text{i.e } n_1 = n_2 = n_3 = \dots \dots \dots = n_N = \frac{1}{N} \times \frac{1}{N} = \dots \dots \dots (xii)$$

*Nicely, we have a result that power consumption in a nano circuit comprising of 'N' number of identical carbon nano resistors arranged in parallel configuration is N times than that of obtained from an arrangement of the same given number of carbon nano resistors in series configuration.*

MIXED GROUPING CONFIGURATION

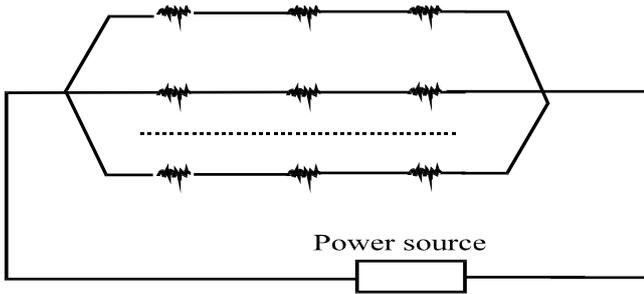


Fig-03

Now, let us consider 'N' number of total carbon nano resistors are arranged in mixed grouping with first row containing  $N_1$ , second row containing  $N_2$  and the third containing  $N_3$  resistors and so on. Therefore power loss across the first row, may be given as,

$$P^{1s} = \frac{2V_{pd}^2 e^2}{h} \frac{1}{\sum_{i=1}^{N_1} n_i^1}$$

Similarly power loss across the second row

$$P^{2s} = \frac{2V_{pd}^2 e^2}{h} \frac{1}{\sum_{i=1}^{N_2} n_i^2}$$

Similarly power loss across the third row

$$P^{3s} = \frac{2V_{pd}^2 e^2}{h} \frac{1}{\sum_{i=1}^{N_3} n_i^3}$$

Note that  $n_i^1, n_i^2, n_i^3, \dots$  represent the quantum states of the carbon nano resistors in the first, second and third rows respectively and so on for other rows too. But, it is to be reminded that these are merely the notations representing the quantum states of the carbon nano resistors and note that  $n_i^2, n_i^3, \dots$  in each row may not run consecutively.

Therefore one can easily calculate the net power loss in the entire arrangement by adding all the above calculated power losses for each row.

i.e  $P_n^M = P^{1s} + P^{2s} + P^{3s} + \dots$

$$= \frac{2V_{pd}^2 e^2}{h} \left( \frac{1}{\sum_{i=1}^{N_1} n_i^1} + \frac{1}{\sum_{i=1}^{N_2} n_i^2} + \frac{1}{\sum_{i=1}^{N_3} n_i^3} + \dots \right) \dots \dots (xiii)$$

If there are exactly 'm' rows of carbon nano resistors, then one can generalize as follows

$$P_n^M = \frac{2V_{pd}^2 e^2}{h} \left( \sum_{k=1}^m \frac{1}{\sum_{i=1}^{N_k} n_i^k} \right) \dots \dots \dots (xiv)$$

Now, for simplicity we consider in particular, when all the resistors are identical in a row (having same quantum state) and writing the notation such as 'k'th row has  $N_k$  number of resistors all having identical quantum state  $n_k$ ; then we can represent the above relation as

$$P_n^M = \frac{2V_{pd}^2 e^2}{h} \left( \sum_{k=1}^m \frac{1}{N_k n_k} \right) \dots \dots \dots (xv)$$

For special case when all the carbon nano resistors are

having same quantum states say 'n' then at once one can reduce the above relation as

$$P_n^M = \frac{2V_{pd}^2 e^2}{hn} \left( \sum_{k=1}^m \frac{1}{N_k} \right) \dots \dots \dots (xvi)$$

Finally, we assume that in each row, number of resistors is the same  $N_k = N$ ,  $P_n^M = \frac{2V_{pd}^2 e^2}{hn} \dots \dots \dots (xvii)$

$$P_n^M = \frac{2V_{pd}^2 e^2}{hn} = \frac{2V_{pd}^2 e^2 N}{hn N^2}$$

$$P_n^M = \frac{N}{N^2} \times \dots \dots \dots (xviii)$$

where  $i=1,2,3,4, \dots$

If there are as many rows as the number of resistors in each row, then  $N' = N$  and one can write

$$P_n^M = P_i = \frac{2V_{pd}^2 e^2}{hn} \dots \dots \dots (xvii)$$

Thus, in the mixed grouping of identical carbon nano resistors in a nano circuit when there are exactly as many rows as the number of carbon nano resistors in each row, then the power loss across the arrangement is identical to that of the simple circuit containing merely a single carbon nano resistor and thus power loss is independent of the number of resistors, behaving as a circuit of single resistor.

EQUIVALENT QUANTUM STATE

Supposing all the carbon nano quantum resistors of different quantum states then for series configuration we have,

$$R_n^s = R_{n_1} + R_{n_2} + R_{n_3} + \dots$$

Or

$$= \frac{h}{2} + \frac{h}{2} + \frac{h}{2} + \dots$$

If 'n' is the equivalent quantum state of the single carbon nano resistor on introducing in the nano circuit such that it replacing all the carbon nano resistors in series, keeping the current same in the circuit then we can write  $\frac{h}{n} = \frac{h}{2} + \frac{h}{2} + \frac{h}{2} + \dots$

Or  $n = 2 + 2 + 2 + \dots + 1 \dots \dots (xix)$

For parallel configuration,

$$\frac{1}{R_n^p} = \frac{1}{R_{n_1}} + \frac{1}{R_{n_2}} + \frac{1}{R_{n_3}} + \dots$$

$$\frac{2e^2}{hn} = \frac{2e^2}{h} \left( \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \dots \right)$$

$$\frac{1}{n} = \left( \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \dots \right) \dots \dots \dots (xix)$$

III. FINDINGS OF THE RESULTS

My approach was totally theoretical supported by experimental background of quantized resistance and quantized current in the nano circuit. Despite of adequate simplicity we have simulated the quantized result for net power consumption in the nano circuit, of course reminding that we should mention, we have totally ignored the internal resistance of the power source as modern technology is adequate to develop such a cell of negligible internal resistance. However, let us have a look on the findings of my

theoretical works in brief .

(1) **Power loss**  $P_n^s = \bar{P}_n^s = \frac{2V_{pd}^2 e^2}{h} \frac{1}{\sum_{i=1}^N}$  , **series configuration**

(2) **Power loss**  $P_n^p = \bar{P}_n^p = \frac{2V_{pd}^2 e^2}{h} \sum_{i=1}^N$  , **parallel configuration**

(3)  $\frac{P_n^p}{P_n^s} = \frac{1}{\sum n_i} \times ;$

*This is why, power loss or consumption for a given set of carbon nano resistors is more in parallel configuration compared to that of in series configuration*

(4) *Nicely, we have a result that power consumption in a nano circuit comprising of 'N' number of identical carbon nano resistors arranged in parallel configuration is N times than that of obtained from an arrangement of the same given number of carbon nano resistors in series configuration.*

(5) **Power loss**  $\bar{P}_n^M = P_n^M = \frac{2V_{pd}^2 e^2}{h} \left( \sum_{k=1}^m \frac{1}{\sum n} \right)$  ,

**mixed grouping**

(6) *If there are as many rows as the number of resistors in each row , then  $N' =$  and one can write*

$$P_n^M = P_i = \frac{2V_{pd}^2 e^2}{hn}$$

*Thus, in the mixed grouping of identical carbon nano resistors in a nano circuit when there are exactly as many rows as the number of carbon nano resistors in each row, then the power loss across the arrangement is identical to that of the simple circuit containing merely a single carbon nano resistor and thus power loss is independent of the number of resistors ,behaving as a circuit of single resistor*

(7)  $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \dots$

Thus more beautiful feelings arise on looking these above two relations which state that ‘The equivalent quantum state of all the carbon nano resistors in series configuration in a circuit is simply the algebraical sum of all the quantum states of those given resistors’ and ‘The reciprocal of equivalent quantum state of all the carbon nano resistors in parallel configuration in a circuit is simply the algebraical sum of the reciprocals of all the quantum states of those given resistors’ .

(8) *We have seen that in all the three configurations the power loss or consumption varies directly as the square of the applied potential difference or E.M.F.*

#### IV. CONCLUSIONS AND REMARKS

We assumed electron in CNT or one dimensional quantum nano resistor has a few restrictions such as  $\sqrt{A} < L$  and the potential well is infinite depth to have quantize nature of resistance and which in turn quantize the current in discrete values. Apart from this ,in practice the internal resistance of the cells in the nano scale set up,

is expected to be much more smaller than external quantized resistance of the carbon nano resistors in normal room temperature because  $\frac{n\hbar}{2e^2} \approx 10^4$  and no internal resistance is expected to be of the order of 1. This in turn expresses that external resistances in the nano circuit in room temperature must far exceed the internal resistance of the cells , hence current is said to be state dependent and strictly speaking current is strongly quantized in the nano circuit comprising of one dimensional carbon nano resistors .If this results holds good , our calculations so far done for extracting those eight results summarized under the heading of ‘FINDING OF THE RESULTS’ might help those researchers for developing further elaborated schemes suitable in many aspects and area for simple one dimensional carbon nano wirese including extension of those results even in complicated composite structure of CNT.

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