International Journal of New Technology and Research (IJNTR)
ISSN:2454-4116, Volume-2, Issue-7, July 2016 Pages 01-03

Non Dependence of Drift Velocity of Electron on Length of Carbon Nanotubes, Dependence on Quantum State and Expression of Current Density

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***Abstract*: The field of carbon nano tubes (CNT) is an active area of research theoretically as well experimentally [1,2]. It is established fact that samples of single wall carbon nano tubes containing tubes with an arm chair wrapping have been produced and exhibit metallic behaviour. In this paper, the quantized value of electrical conductivity is used to show theoretically that the drift velocity of electron is independent of the length of CNT and 'V_d', drift velocity varies inversely as the quantum state. From which we can express current density of electrons in CNT and can show current density too depends on quantum state.**

***Index Terms* : Driftvelocity, carbon nano tubes, electrical conductivity, potential well**

I. Introduction

it has been so far shown [See,for instance ref. [1], when CNT is extremely narrow (Thin) *i.e* for a very thiin conductor

$A^{\frac{1}{2}} \ll$

L , where b' A' is the area of cross section and d' L' is the

en we start with an ideal one dimensional potential well of length 'L' , assuming infinite depth of the well, then the well known quantized energy is given by

$E_n = \frac{n^2\pi^2h^2}{2mL^2}$------------- (1)

ONEDIMENSIONAL WELL (FIG-01)

Many experiments show that CNT are ballistic conductors [3]; since, the motion of electrons is ballistic, v_n is the velocity of the electron

$$v_n = \frac{nh}{2ml} \text{ ------------------------(2)}$$

Introducing transit time of the electron in nano tube

$$\tau_n = \int_0^L \frac{dX}{V_n} = \frac{L}{v_n}$$

$$\tau_n = 2m\frac{L^2}{nh} - - - - (3)$$

For one electron system ,electron spatial density

$$N_n = \frac{1}{AL} \text{ ....(4)}$$

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With the aid of equation (2), (3) & (4) it has been so far established theoretically

$$\sigma_n = \frac{2Le^2}{Anh} \text{ ....(5)}$$

Equations (1), (2), (3), (4) & (5) have been referred to [1]
Now, with the help of quantized value of the conductivity, we here make an attempt to establish that ' V_d' drift velocity of electron due to an external electric field 'E' is independent of the length of the carbon nano tube.

II. THEORY

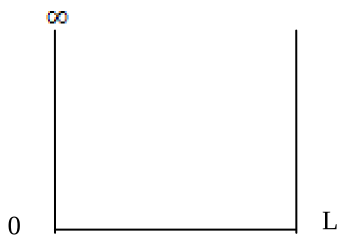

FIG-01 onedimensional well

Respecting the quantum behavior of the conductivity, we denote 'σ' as $\sigma_n = \frac{2Le^2}{Anh}$L

Note, 'σ_n' depends on the length & area of the cross section of the tube. Thus, for a conducting CNT 'σ_n' and hence 'ρ_n' resistivity depends on its dimension , whereas for an ordinary conductor 'σ_n' & 'ρ_n' are independent of the dimension of the given conductor.
Equation (5) shows conductivity is quantized, so does the resistance also

$$R_n = \rho_n \frac{L}{A} = \frac{Anh}{2Le^2} \times \frac{L}{A}$$

This value agrees with first two values for n=1 and n=2 observed in ref[4]

$$R_n = \frac{nh}{2e^2} - - - - - - - - - - - - - - - - (6)$$

So ;the resistance no longer depends upon the dimensional aspect such as on length and its area of cross section but for an ordinary conductor $R_n \alpha L$

$\alpha \frac{1}{A}$

It is here the beautiful nature manifested by the quantum nano wire unlike ordinary conductor, draws attention of the Researchers for further fruitful investigations in quantum nano technological field. Further more ,under the effect of electric field E , the free electron in the wire get accelerated and acquire velocity component in a direction opposite to the direction of electric field in addition to their thermal random velocity. However, the gain in velocity of an electron (e) due to electric field takes place only for a short duration of time as the electron accelerates, it gets scattered or deflected on suffering collision with the positive ions in the conductors.
The short time for which an electron get accelerated before it under goes a collision is called relaxation time. If an electron having initial random thermal velocity $\mathbf{U_1}$ (Bold letters indicate vectors) get accelerated for a time τ_1,then it will attain a velocity V_1,

$$\mathbf{V_1 = U_1 + a}\,\tau_1$$

For many electron system

$$\mathbf{V_2 = U_2 + a}\,\tau_2$$
$$\mathbf{V_3 = U_3 + a}\,\tau_3$$

$$\mathbf{V_n = U_n + a}\,\tau_n$$

Now,by definition of drift velocity,
Drift velocity is the average velocity with which free electrons get drifted under the influence of an external electric field.

$$\mathbf{V_d} = \frac{\mathbf{V_1 + V_2 + V_3}}{\mathbf{n}}$$

$$\mathbf{V_d} = \frac{(\mathbf{U_1 + a}\,\tau_1) + (\mathbf{U_2 + a}\,\tau 2) + (\mathbf{U_3 + a}\,\tau_3)}{n}$$

$$\mathbf{V_d} = \frac{\mathbf{U_1 + U_2 + U_3 \ldots\ldots U_n) + a}(\tau_1 + \tau_2 + \tau_3 + \ldots\ldots \tau_n)}{n}$$

$$\mathbf{V_d = 0 + a}\tau$$

Here, 'τ' average relaxation time and in an conductor random thermal velocities of electrons get cancelled due to random direction of thermal velocities

$$\frac{\vec{U}_1 + \vec{u}_2 + \ldots\ldots\ldots\ldots \vec{U}_n}{n} = 0$$

$$\text{Vd} = -\frac{eE\,\tau}{m} \text{-------------------(7)}$$

It is to be noted the same result given by equation (7) does not hold for single electron system,which involves a factor $\frac{1}{2}$, that comes out as aresult of the average of final and initial velocity of the electron between two successive collisions, which has been derived by simple mathematical calculation as follows

According to free electron model, the electron in a solid move freely. If free time *i.e* time taken between two successive collision be 'τ', *mean free path* ' λ ' then,

$\tau = \frac{\lambda}{v}$, If the applied field on electron of charge $-e$ is '$\vec{E}$', *then the equation of motion of the* electron

$$-e\,\vec{E} = m\frac{d^2x}{dt^2} \text{---------------------------------------(8)}$$

Integrating the above equation,taking magnitude only

$$v = \dot{x} = e\frac{E}{m}t + C,$$

' C' is the costant of intregation
When 't'=τ then $v = v_{max}$
. At t=0 , C=0 as $\dot{x}_0 = 0$,Maximum velocity of electron corresponds to a time duration 'τ' as the electon start with a fresh velocity after a collision.,since the electron moves with uniform acceleration during the time interval'τ' ,average velocity between successive clollision

$$v_d = \frac{v_{max}+0}{2} = e\frac{E}{m}\tau\frac{1}{2} \text{ -------------------------------(9)}$$

We could see a factor ' $\frac{1}{2}$' in the expression, since the drift velocity has to be an average velocity of the journey not the terminal velocity between successive collision.
So , this idea of drift velocity can be extended to many electron system as well, instead of taking single electron system as so far, we have considered only single electron in one dimensional well of CNT.For a system of 'n'electrons-

$$\mathbf{V}_d = (\mathbf{u_1}+\mathbf{v_1}) + (\boldsymbol{u_2} + \boldsymbol{v_2}) + \text{-----------------} + (\boldsymbol{u_n} + \boldsymbol{v_n})/2n$$

Note bold letters have been used to denote vectors

$$\mathbf{V_d} = (\boldsymbol{u_1} + \boldsymbol{u_2} + \boldsymbol{u_3} + \text{---------}\boldsymbol{u_n})/2n + (\boldsymbol{v_1} + \boldsymbol{v_2} \text{-+------} + \boldsymbol{v_n})/2n$$
$$= 0 + \vec{a}\,(\tau_1 + \tau_2 + \tau_3 + \cdots \ldots\ldots\ldots \tau_n)/2n$$

'τ' is here referred to average relaxation time

$$\tau = \frac{\tau_1+\tau_2+\tau_3+\cdots+\tau_n}{n}$$

$$\overrightarrow{v_d} = \vec{a}\,\frac{\tau}{2} = -e\frac{\vec{E}}{m}\frac{\tau}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

So, the correct value of drift velocity is expected to be given by equation(10) for n electron system

Now, equation (9) and(10) gives same drift velocity. But many authors use drift velocity as

$$\overrightarrow{v_d} = -e\frac{\vec{E}}{m}\tau \text{ as in ref[5,6],pg no 184}$$

Since ,current through the nano tube can be given ref[7],
$i = \overline{N}_n A\, v_d e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$

'$\overline{N}_n$' *is the electron density*
Current density 'J' can now be given as
$J = \overline{N}_n v_d e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$

After substituting the value of 'v_d' in the above equation ,we have………

$$J = \frac{\overline{N}_n e^2 E\tau}{2m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\mathbf{J = \sigma E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

From equatios , (13)&(14), we can write as follows

$\rho = \frac{1}{\sigma} = \frac{2m}{\overline{N}_n e^2 \tau}$ ………………………………………….(15)

From relations-(12)&(14) ,we can write

$v_d = \frac{E}{\overline{N}_n \rho e} = \frac{V_{p.d}}{L\overline{N}_n \rho e}$………………………………… (16)

$E = \frac{V_{p.d}}{L}$, '$V_{p.d}$' is potential difference across the carbon nano tube

Now, using the quantized value of 'ρ' in the above equation, we have

$v_d = \frac{V_{p.d}}{L N_n e} X \frac{2Le^2}{Anh}$

$V_{drift} = \frac{2V_{p.d}e}{Anh N_n}$ …………………………………(17)

Recalling 'n' as quantum state, very interestingly we can compare that in conductor drift velocity depends on length but in nano- tube, 'v_d' (drift velocity) does not depnend on length. It is however a consequence of our assumption [1], for thin wire $A^{\frac{1}{2}} \ll L$ at the starting of our theoretical work. If $A^{\frac{1}{2}} \approx L$, *then* the above result given by equation (17) is likely to be ceased and is no longer valid.

We can see drift velocity of electron also depends on its quantum state 'n' given by equation (17), also it is clear that the drift velocity of electron in carbon nano tubes varies inversely as the area of cross section 'A' of the carbon nano tube. It is to be reminded that in ordinary conductor, the expression of drift velocity does not contain 'A' .

It is now convenient to express current density 'J' in terms of the electric t field 'E' ,

Since; $I = \overline{N}_n A v_d e$ …………………………………. (18)

$J = \overline{N}_n v_d e$

Now, substituting the quantized value of 'v_d' in the above expression' we have

$J = \frac{2V_{p.d}e^2}{Anh}$ ……………………………………………. (19)

The current density is found to varies inversely as the area of cross section of the nano tube and also varies inversely on the quantum state 'n'.

III. Conclusion

Thus, the ordinary conductor and CNT differ in many respects such as in the conductivity, resistivity, resistance and the drift velocity of the electron etc. ρ_n or σ_n depends on the dimension for a CNT whereas for ordinary conductor ρ_n or σ_n does not depend on the dimension of the conductor. Thus, ρ_n or σ_n in carbon nano tube are no longer constants for a given nano-tube unlike as that of the case in ordinary conductors .Also, V_d (drift velocity) of the electron in carbon nano- tube is independent of the length of the CNT whereas in ordinary conductor drift velocity depends on its length.

In addition to this, the drift velocity of electron in CNT depends on its area of cross section and its quantum state but ordinary conductor classically does not contain its area of cross section and quantum state in the expression of drift velocity.

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The name of the Author of this research paper is Ashfuz Zaman . After completion of H.S.L.C he took admission in to B. N College, Dhubri. In the H.S(Sc.) 2nd year he appeared in final Exam. at t J.M H.S School, Mankachar. His inherent interest in Physics inspired him to get admitted him self at GOalpara College, GOalpara in 2001 and he was given Physics honours from which came out successfully securing first class firs position in B.Sc final Exam. That, he was further driven by natural interest and curiosity that motivated him to persue his post Graduate degree from Gauhati University and seared First class .

Again in the next year he completed his B.Ed from Govt. B.T college, Goalpara.

Tthereafter he joined Assistant Professor at Maryam Ajmal College of Science and Technology and Ajmal Boys College of Arts Commerce and Science, Hojai. Serving there for two years he shifted to Goalpara and join a pioneer Junior College namely Science Academy, Bhalukdubi. Rendering his service for two years he join Mahatma Gandhi Junior College, Goalpara and joined a Coaching Institute at the head quarter, Goalpara. Being selected at several coaching Institutes at Guwahati he appeared in SLET(N.E) Exam. in2015 and qualified the Exam. being North East topper. At present he is going to publish his very first book "SPECIAL THEORY OF RELATIVITY " for B.Sc Major and has carried out some research work which is still on progress. Further he would like to devote him self in the field of applied Physics for completion of his Research works so as to complete Ph.D degree.