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Stability Assessment of a Manufacturing Process Using Grey Relationship Theory Based on Two Unsorted Data Series

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***Abstract*—Based on grey system theory, which is used to analyze two unsorted data series produced during the manufacturing process, the stability of a manufacturing system can be evaluated. First, two unsorted data series with the original order maintained are acquired from a manufacturing system. The grey relationship of the two data series without sorting can be established based on the distribution features of data series. Grey confidence level is calculated to evaluate the stability of a manufacturing process dynamically. A simulation experiment and an actual case were studied by analyzing two unsorted data series produced during the manufacturing process to evaluate the stability of manufacturing systems. In the simulation experiment, a normal distribution with unsorted data sequence was added with a linear error distribution sequence with the same number of data points. The result of simulation experiment showed that the grey confidence level P = 69.5% < 90% when the weight f =0.5. Therefore, two unsorted data series did not have the same properties, indicating that the manufacturing process was not stable. In the actual case, a sine function distribution was added to a triangular distribution with the same number of data points to generate a systematic error. The result of actual case showed that the grey confidence level P = 89.5% < 90% when the weight f =0.5, indicating that the manufacturing process was not stable. The stability assessment model is proven to be able to be used to analyze the system property of two unsorted data series under the condition of poor information, which is beyond the constraints of grey relational sequence space in original grey system theory and is the expansion of the grey system theory.**

***Index Terms*—stability assessment, grey relationship, unsorted data series, manufacturing system, grey confidence level**

I. INTRODUCTION

In a mechanical manufacturing system, the uncertainty in product components has a significant effect on the performance and reliability of the mechanical system. Thus, whether in academia or industry, the quality control problems in manufacturing process draw much attention [1]–[4]. Based on the mechanical manufacturing theory, the process stability of a manufacturing system should be assessed and predicted in real time to identify unstable factors in the manufacturing process, so that appropriate measures can be taken as soon as possible to meet quality requirements [5]–[7].

In the existing literatures, most approaches for analyzing the stability of a manufacturing process rely on traditional statistical methods, which assume that the errors of a manufacturing system are normally distributed [8]. However, the attributes of manufacturing systems are generally more complex than a normal distribution, instead following distributions such as a typical or atypical distribution. Furthermore, unknown attributes may also be present in the systems [9]. In these cases, the use of traditional statistical methods to analyze the stability of a system is less reliable.

In recent years, the research on poor information theory becomes the hot issue. Poor information refers to that the information is poor of severely poor. For example, the probability distribution of the whole system is unknown or very complicated in the evaluation process of mechanical system and maybe there is only small data sample for reference, which belongs to the questions of poor information.

The concept of grey correlation degree can be used to solve the problem of analyzing the uncertain systems with poor information, which has no any requirement for the probability distribution of system and little output data can be allowed for the system. In fact, there are many uncertain systems with poor information in practical engineering. Especially in the modern science and technology research, such as, a new type spaceflight test, the high cost destructive test and special weapons and equipment system, the priori information is great shortage, which is difficult to solve using classical statistical theory and belongs to the category of the poor information system theory [10].

However, it is usually required at least three systems to analyze the relative relationship of the system properties based on the poor information theory. At present, great progress has been made in using the grey relationship concept in grey system theory to analyze problems [11]–[13].

The stability of one manufacturing process is analyzed based on the system properties of two data series using the grey relational concept, in which no special requirements for the probability distribution are imposed for the manufacturing process. The grey relationship between the two data series maintaining the original order is calculated based on the probability distribution features of data series, which is then utilized to evaluate the stability of the manufacturing process. So the stable operation of a manufacturing system can be described to improve the

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uncertainty of products and enhance product quality.

II. Methods

A. Unsorted Grey Relationship for a Manufacturing Process

Based on the concept of grey relationship, the properties of a system can be expressed by mapping, which typically include parameters of the system or data produced by the system. In a manufacturing process, two data series $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ can be given as follows:

$$\boldsymbol{X}_i = (x_i(1), x_i(2), ..., x_i(k), ..., x_i(K)) \tag{1}$$

$$\boldsymbol{X}_j = (x_j(1), x_j(2), ..., x_j(k), ..., x_j(K)) \tag{2}$$

where i and j are the sequence numbers of data series; k is the sequence number of data point in the unsorted data series; K is the total number of data in data series; $x_i(k)$ and $x_j(k)$ are the kth data in the unsorted data series.

$x_0(k)$ is the mean value of data series, which can be calculated by

$$x_0(k) = f(x_i(k), x_j(k)) = (\sum_{k=1}^{K} x_i(k) + \sum_{k=1}^{K} x_j(k)) / (2K) \tag{3}$$

$$\boldsymbol{X}_0 = (x_0(1), x_0(2), ..., x_0(k), ..., x_0(K)) \tag{4}$$

where $\boldsymbol{X}_0$ is a reference sequence based on the average of the data series.

For any $k \in K$, the elements in the sequence $\boldsymbol{X}_0$ can be constant values based on the principle of minimum information.

$$x_0(k) = \boldsymbol{X}_0 = x_i(1) \tag{5}$$

For $\boldsymbol{X}_h \in (\boldsymbol{X}_i, \boldsymbol{X}_j)$ and $h \in (i, j)$, the grey correlation degree can be calculated by

$$\gamma_{0h} = \gamma(X_0, X_h) = \frac{1}{K} \sum_{k=1}^{K} \gamma(x_0(k), x_h(k)) \tag{6}$$

Assume the distinguishing coefficient $\xi \in (0, 1]$, so the grey relation coefficient can be obtained by

$$\gamma(x_0(k), x_h(k)) = \frac{\Delta_{\min} + \xi \Delta_{\max}}{\Delta_{0h}(k) + \xi \Delta_{\max}}; \; k = 1, 2, ..., K \tag{7}$$

The grey difference information is thus given by

$$\Delta_{0h}(k) = |x_h(k) - x_0(k)| \tag{8}$$

The range can be expressed as

$$\Delta_{\min} = \min_h \min_k \Delta_{0h}(k) \tag{9}$$

$$\Delta_{\max} = \max_h \max_k \Delta_{0h}(k) \tag{10}$$

The grey difference between the two data sequences $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ is defined as

$$d_{ij} = |\gamma_{0i} - \gamma_{0j}| \tag{11}$$

$$r_{ij} = 1 - d_{ij} \tag{12}$$

where r_{ij} is the similarity coefficient between the data sequences $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ based on grey relational degree.

The grey similarity matrix is also known as the grey relation attribute or the grey relationship, and $0 \le r_{ij} \le 1$.

$$\boldsymbol{R} = \{r_{ij}\} = \begin{bmatrix} r_{ii} & r_{ij} \\ r_{ji} & r_{jj} \end{bmatrix} = \begin{bmatrix} 1 & r_{ij} \\ r_{ji} & 1 \end{bmatrix} \tag{13}$$

$\xi \in (0, 1]$, there is always only one real number $d_{\max} = d_{ij\max}$ that satisfies that $d_{ij} \le d_{\max}$, where $d_{\max}$ is defined as the maximum grey difference and the corresponding ξ is defined as the optimal resolution coefficient based on the maximum grey difference.

The attribute weights based on the grey relationship between the two data sequences $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ can be described by

$$f_{ij} = \begin{cases} 1 - d_{\max} / \eta; & d_{\max} \in [0, \eta] \\ 0; & d_{\max} \in [\eta, 1] \end{cases} \tag{14}$$

where f_{ij} is the attribute weight, $f_{ij} \in [0, 1]$; η is a parameter, $\eta \in [0, 1]$.

Based on the albino and symmetry principles, λ is considered as the representative of a true yuan under the condition of a given criteria, if there is no reason to deny that λ is a true yuan. Assume that $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are known and $\lambda \in [0, 1]$, $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ will have the same properties if there is a mapping that $f_{ij} \ge \lambda$. When $f_{ij} = \lambda = 0.5$, $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are considered to have the same properties here.

Assume that $\eta \in [0, 0.5]$, then

$$d_{\max} = (1 - f_{ij})\eta \tag{15}$$

$$P = P_{ij} = 1 - (1 - \lambda)\eta = (1 - 0.5\eta) \times 100\% \tag{16}$$

where P is the grey confidence level, which describes the possibility that $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ have the same property.

If only a system X is studied as follows:

$$X = \{x(u) \mid u = (1, 2, 3, ..., 2n)\} \tag{17}$$

Then, X is divided into two subsystems, $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ by

$$X_i = \{x(u) \mid u = (1, 2, ..., n)\} = \{x_i(k) \mid k \in K\} \tag{18}$$

$$X_j = \{x(u) \mid u = (n+1, ..., 2n)\} = \{x_j(k) \mid k \in K\} \tag{19}$$

Grey relationship, therefore, can be used to study the attribute problem of any system.

III. Evaluate Stability of Manufacturing Process

For some manufacturing systems with specific properties, the system properties will not change during the normal manufacturing process. One quality parameter of machinery products, such as dimension error, roundness, roughness, ect., can be considered as a random variable following a particular ideal distribution with an ideal eigenvalue. For example, the eigenvalues of the normal distribution can be described by its mathematical expectation and standard deviation. The eigenvalue of the Rayleigh distribution is its standard deviation, and the eigenvalues of uniform distribution are the lower limit value and the upper limit value of independent variable. Due to the uncertainty in the manufacturing process, some types of disturbances occur during processing, which causes that the distribution of quality parameters deviates from the ideal distribution and the corresponding eigenvalue (i.e., the properties of the system change). That is a feature of instability in manufacturing process.

Based on the concept of grey relationship, the more similar the unsorted data sequences $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are, then the bigger the value of the grey confidence level will be and the more stable the manufacturing process is. Conversely, the smaller the grey confidence level is, the less stable the manufacturing process is. In an actual test on the stability of a manufacturing process, if the unsorted data sequence $\boldsymbol{X}_h \in (\boldsymbol{X}_i, \boldsymbol{X}_j)$ and $h \in$ (i, j), assume that $f = f_{ij} = 0.5$. Whether the manufacturing process is stable is determined by calculating the grey confidence level. If the grey confidence level P is above 90%, X_i and X_j can be considered to have the same properties and the manufacturing process is stable. Otherwise, X_i and X_j have different properties, and the manufacturing process is not stable.

It should be noted that the grey relationship for unsorted data series can be used to test the different parameters in a system as well as the difference between all types of parameters among different systems and within a single system.

IV. EXPERIMENTAL INVESTIGATION

A. Simulation Experiment

A simulation experiment is used to investigate the confidence level of the grey relationship between the two unsorted data sequences to validate the theory described above.

To simulate a normal distribution with a mathematical expectation E = 0 and standard deviation s = 0.01, a total of 60 data points are generated as shown in Fig.1. In the computer simulation process, the output of the simulation is the output of a manufacturing process with a normal distribution. So the ideal distribution of the parameters (i.e., random variables) of the product manufactured is a normal distribution.

The unsorted data sequence is then added with a linear error distribution sequence with the same number of data points to generate a systematic error. The first element of the arithmetic data sequence is 1, and the tolerance is 1. The arithmetic data sequence is shown in Fig.2, and a new unsorted data sequence X is shown in Fig.3.

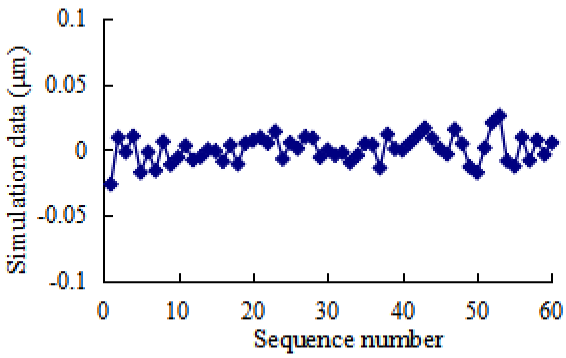

Fig.1 Simulation results for a normal distribution

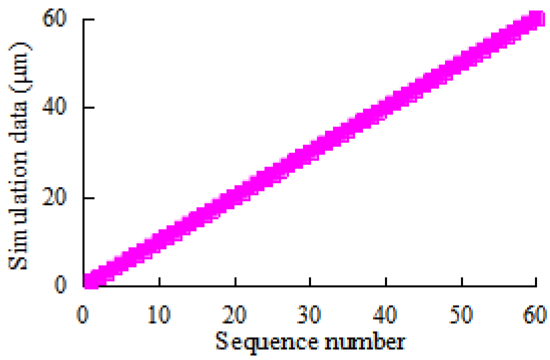

Fig. 2 Simulation data sample with a linear error

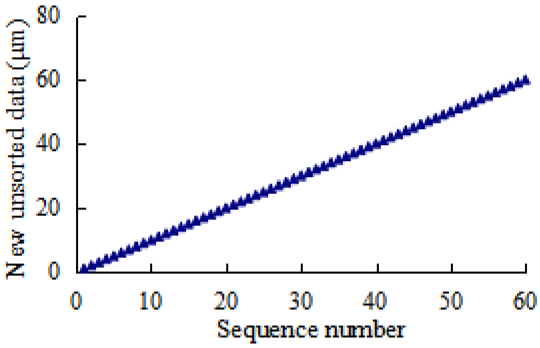

Fig. 3 The generated data sample for a normal distribution with a linear error

Assuming that the first 30 data points constitute the data sequence $\boldsymbol{X}_1$ and the last 30 data points constitute the data sequence $\boldsymbol{X}_2$, where h=1 and 2, m=2, and n=30. The reference sequence can be given by

$$x_0 = x_1(1)$$

The calculation results for the unsorted grey relationship are as follows:

x_0=0.97389; ξ^*=0.3001; γ_{01}=0.5952; γ_{02}=0.2903; $d_{12}{}^*$=0.304.

When f =0.5, the grey confidence level P = 69.5% < 90%. Therefore, when the weight f =0.5, $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ do not have the same properties, which indicates that the manufacturing process is not stable. Figure 3 shows that due to the influence of the linear error, the normal distribution of the output of the manufacturing process is almost completely replaced by the linear error distribution. As a result, the feasibility of testing the stability of a manufacturing process using the grey relationship is verified.

B. Practical case

In a practical engineering application, the Monte Carlo method can be used to simulate the output of a manufacturing process that is known to follow a sine function distribution with given characteristic values (e.g., a basic value equals to 5 and an amplitude A equals to 1). 60 data points are generated as shown in Fig.4.

The sine function distribution is then added to a triangular distribution with the same number of data points to generate a systematic error. The characteristic value of the triangular distribution is 5, the range is (-1, 1). The simulation results are shown in Fig.5, and a new unsorted data sequence X is shown in Fig.6.

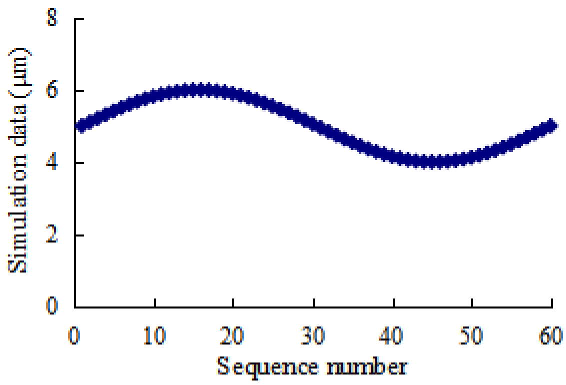

Fig. 4 Simulation results for a sine function distribution

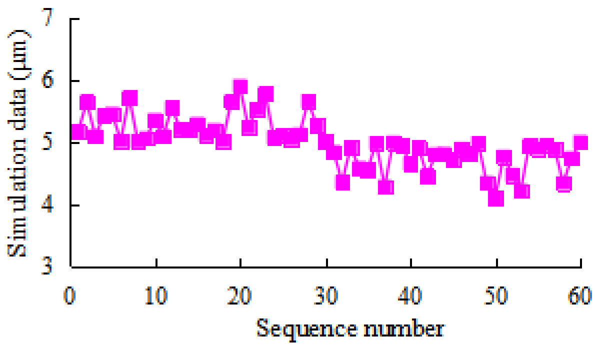

Fig. 5 Simulation results for a triangular distribution

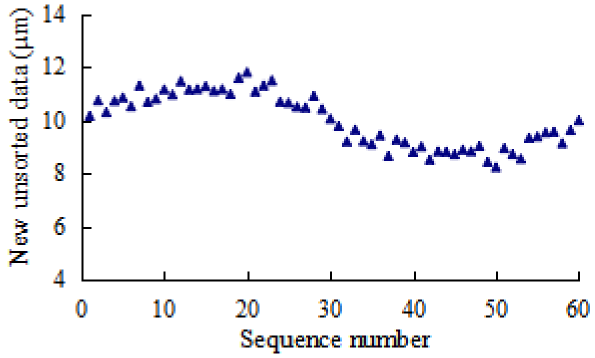

Fig. 6 Stability of a sine function distribution with a triangular distribution

Assuming that the first 30 data points are the data sequence $\boldsymbol{X}_1$ and the last 30 data points are the data sequence $\boldsymbol{X}_2$, where h=1 and 2, m=2, and n=30. Then the reference sequence can be described as

$$x_0=x_1(1)$$

The results of the unsorted grey relationship are as follows: x_0=10.16456; ξ^*=0.3001; γ_{01}=0.4808; γ_{02}=0.3759; d_{12}^*=0.104.

When f = 0.5, the grey confidence level P = 89.5% < 90%. Therefore, when the weight f = 0.5, $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ do not have the same properties, indicating that the manufacturing process is not stable. As shown in Figure 6, the sine function distribution of the output for the manufacturing process is affected by the triangular distribution. The diagram contains characteristics of both the sine distribution and the triangular distribution.

V. Conclusion

The stability of a manufacturing process was evaluated using the grey relationship between two data sequences, with no special requirements for the probability distribution of data sequences. The grey confidence level of two unsorted sequence was calculated to identify the process instability phenomenon of the manufacturing system. Increasing the stability of the manufacturing process will improve uncertainties in product quality and enhance the overall product quality.

A simulation experiment and an actual case study were used to demonstrate that the proposed grey relationship analysis could accurately describe the stability of a manufacturing process. As a result, the feasibility of testing the stability of a manufacturing process using the grey relationship analysis of an unsorted data sequence was verified. The unsorted grey relationship was used to dynamically evaluate the stability of a manufacturing process to real-time monitor for a manufacturing process.

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