

Synchronization, Control and Stability of Fractional Order Hyperchaotic Systems in The Context of The Generalized Memory

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Abstract— In the article offered fractional kinetic model of networks with generalized memory. On the basis of fractional kinetic model network with hyperchaotic systems, embedded in a percolation structure, realized task topologically synchronization. While tracing control and stability. Criterion – “proximity” capture average return time Poincare. Shows a visualization of results.

Index Terms—fractional kinetic model, topological synchronization, generalize memory, Poincare return time.

I. INTRODUCTION

The most control problems of modern philosophy of physics are [1, 2]: problem of singularity (space and time-mega world); complementarily problem (determinism and causality-micro world); the problem of self-organization (chaos and order - macro world); problem of the relationship of the empirical and the speculative knowledge.

Accounting and the impact of these problems is considerable interest in the context of the analyses and synthesis of fractional kinetic Network models synchronization, control and stability.

Naturally that this paradigm is permitted under the synergetic approach while remaining within the physics Research Area.

To continue publishing in this article provides a structure “Topological synchronization Tracing control stability” is permitted in terms of the kinetic model of the network of fractional order.

An important feature of the model is to determine the parameters of the percolation at the micro level. Kinetic model realized in the space of Poincare fractional order [3].

II. NOTATION AND PRELIMINARIES

A. *n*-dimensional-fractional-order chaotic system.

Consider the following *n*-dimensional-fractional-order chaotic system

$$D^q X = F(X, X_0, \theta) \quad (1)$$

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where $X = (x_1, x_2, \dots, x_n)^T \in R^n$ denotes the *n*-dimensional state vector of the original system; X_0 - represents the system initial state, $q = (q_1, q_2, \dots, q_n)^T \in R^n$ is a set of fractional order of the original system, and $\theta = (\theta_1, \theta_2, \dots, \theta_D)^T \in R^D$ is the value of original system parameters.

Let the fractional-order derivative of the function $f(t)$ in the Caputo sense is defined as:

$$D^q f(t) = J^{m-q} f^{(m)}(t). \quad (2)$$

Here, q is the fractional order, m is an integer that satisfies $m-1 \leq q < m$, $f^{(m)}(t)$ is the ordinary m th derivative of f , and J^μ is the Riemann-Liouville integral operator of order $\mu > 0$, defined by

$$J^\mu g(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} g(\tau) d\tau, \quad (3)$$

where $\Gamma(\cdot)$ denotes the gamma function. A particularly important case in many engineering applications is $0 < q < 1$. In this situation, Eq. (2) together with Eq. (3)

$$D_*^q f(t) = \frac{1}{\Gamma(1-q)} \int_0^t (t-\tau)^{-q} f'(\tau) d\tau \quad (4)$$

The operator D_*^q is often called “ q th-order Caputo differential operator” and will be used throughout the paper.

B. Network model

In this paper, we consider a dynamical network consisting of N identical nodes with nonlinear couplings, in which each node is a *n*-dimensional system obeying the following fractional kinetic equations [4]:

$$D_*^q x = Ax + f(x) \quad (5)$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in R^n$ represents the state vector, $A \in R^{n \times n}$ is a constant matrix, and $f: R^n \rightarrow R^n$ represents the nonlinear part of the oscillator and is assumed to be smooth enough. $q = (q_1, q_2, \dots, q_n)^T$ indicates the fractional orders with all $q_i \in (0,1)$ and

$D_*^q x = (D_*^{q_1} x_1, D_*^{q_2} x_2, \dots, D_*^{q_n} x_n)^T$. We call system (4) a

commensurate order system for $q_1 = q_2 = \dots = q_n$, otherwise system (4) is called an incommensurate order system.

The entire network is a system of nN FDEs particular, the state equations are:

$$D_*^q x_i = Ax_i + f(s(t)), \quad i = 1, 2, \dots, N \quad (6)$$

where $x_i \in R^n$ describes the state of the i -th node and $s(t) \in R^n$ is the coupling signal. These unit systems are nonlinearly coupled by the function

$$s(t) = h(x_1, x_2, \dots, x_N) \quad (7)$$

where $h: R^{nN} \rightarrow R^n$ integrates the state of all units forming a complex network. It should be noted that the integrated function h can be designed in either linear form

$$\left(\text{e.g., } h(x_1, x_2, \dots, x_N) = \frac{1}{N} \sum_{j=1}^N x_j \right)$$

$$\left(\text{e.g., } h(x_1, x_2, \dots, x_N) = \frac{1}{N} \sum_{j=1}^N g(x_j) \right)$$

with smooth function $g: R^n \rightarrow R^n$. From the view of synchronization, both kinds of function h would achieve network synchronization, but with different synchronized state. Therefore, for simplicity, we shall later on restrict ourselves to the linear case, i.e., to linear integrated function h . For such networks, the state equations can be explicitly written as

$$D_*^q x_i = Ax_i + f\left(\frac{\varepsilon}{N} \sum_{j=1}^N x_j\right), \quad i = 1, 2, \dots, N, \quad (8)$$

where $\varepsilon \in R$ is the coupling strength.

C. Topology of fractional-order space

Definition 1. The number is called as a metric order of a compact A

$$k = \lim(-\ln N_A(\varepsilon) / \ln \varepsilon) \quad (9)$$

where ε - the sphere of radius ε ; $N(\varepsilon)$ - number of spheres in a final sub covering of a set.

The lower bound of metric orders for all metrics of a compact A (called by metric dimension) is equal his Lebesgue to dimension.

However it appeared that the metric order entered in [5], coincides with the lower side the fractal dimension of Hausdorff-Bezikovitch defined in the terms "box-counting".

Takes place

Theorem 1 [5]. For any compact metric space X .

$$\dim X = \inf \left\{ \lim_{\varepsilon \rightarrow 0} \frac{\log N_{\varepsilon,d}(X)}{-\log \varepsilon} : d \text{ is a metric on } X \right\},$$

where

$$N_{\varepsilon,d}(X) = \min \left\{ |U| : U \text{ is a finite open covering of } X \text{ with mesh } \leq \varepsilon \right\}.$$

From here (X, d_f) - compact fractal metric space with dimension d_f .

Here it is important to note that at the description of properties of systems with fractional structure it is impossible to use representation of Euclidean geometry. There is a need of the analysis of these processes for terms of geometry of fractional dimension.

Remark. In [6] presented results of communication of a fractional integrodifferentiation (in Rimana-Liouville or Gryunvalda-Letnikov's terms) with Koch's curves.

It is noted that *biunique communication between fractals and fractional operators does not exist*: fractals can be generated and described without use of fractional operations, and defined the fractional operator not necessarily generates defined (unambiguously with it connected) fractal process or fractal variety.

However use of fractional operations allows generating other fractal process (variety) which fractal dimension is connected with an indicator of a fractional integrodifferentiation a linear ratio on the basis of the set fractal process (variety).

In [6] fractional integrals of Riman-Liouville are understood as integrals on space of fractional dimension. Thus the indicator of integration is connected with dimension of space an unambiguous ratio.

In this regard consideration of dimension of chaotic systems of a fractional order causes interest. So, in [9] was noted that dimension of such systems can be defined by the sum of fractional exponents Σ , and $\Sigma < 3$ is the most effective.

Let the chaotic fractional system of Lorentz take place [7]:

$$\frac{d^\alpha}{dt^\alpha} x = \sigma(y - x), \quad \frac{d^\beta}{dt^\beta} y = \rho x - y - xz', \quad \frac{d^\gamma}{dt^\gamma} z = xy - bz \quad (10)$$

here $\sigma = 10, \rho = 28, b = 8/3; 0 < \alpha, \beta, \gamma \leq 1, r \geq 1$.

Then fractional dimension of system of the equations (6) will have an appearance [9]:

$$\alpha + \beta + \gamma = \Sigma \quad (11)$$

So, for example, for Lorentz's system with fractional exponents $\alpha = \beta = \gamma = 0.99$, effective dimension $\Sigma = 2.97$.

This, in the context of fractional dynamics let \tilde{X} - any set of nonlinear physical systems, A^α - a subset of a set \tilde{X} of

systems of a fractional order with memory $A^\alpha \subset \tilde{X}$. Then a triad $(\tilde{X}, A^\alpha, \Sigma)$ – compact fractional metric space with dimension Σ .

Let's designate $W \in (X, d_f)$. On the basis [8] and remarks $(\tilde{X}, A^\alpha, \Sigma) \subset W$.

Let's consider transformation W at an angle of communications of average time of return of Poincare $\langle \tau \rangle$ with d_f and “residual” general memory (GM).

$$\text{Here } g : \langle \tau \rangle \Rightarrow d_f l : d_f \Rightarrow GM, \chi : \langle \tau \rangle \Rightarrow (g, l).$$

From here $U \in (X, \langle \tau \rangle)$ – the generalized compact metric space of Poincare with dimension $\langle \tau \rangle$.

D. Generalized systems with memory

Let $(\tilde{X}, A^\alpha, \Sigma) \subset W$; Z the set of all integers [9]:

$$R_{\geq 0} = [0, \infty), R_{\leq 0} = (-\infty, 0), Z_{\geq 0} = \{1, 2, \dots\}, \text{ and } Z_{\leq 0} = \{0, -1, -2, \dots\}.$$

Definition 2. Let $(\tilde{X}, A^\alpha, \Sigma) \subset W$ be GM. $GM \subseteq R \times Z$ is called a compact generalized memory of [4]:

$$GM = Q_{\geq 0} \cup Q_{\leq 0}, \quad (12)$$

where: $Q_{\geq 0} = \bigcup_{j=0}^{j-1} ([t_j, t_{j+1}], j)$

and: $Q_{\leq 0} = \bigcup_{k=1}^k ([s_k, s_{k-1}], -k + 1)$

for same finite of observed:

$$s_k \leq \dots \leq s_i \leq s_0 = 0 = t_0 \leq t_1 \leq \dots \leq t_j.$$

III. TOPOLOGICAL SYNCHRONIZATION

Unlike traditional methods of synchronization in [10] proposed the concept of topological synchronization of coupled chaotic systems.

Definition 3 [10]. Two systems are topologically synchronized, if Poincare return times behave a similar way.

Thus match the dimensions of these two systems – a necessary condition for the topological synchronization: well indicates the “average similarity” [10]. It is known that the

synchronization feature is the preservation of a certain frequency: in this case, is the relationship between the Poincare return times.

Invariance of these ratios, are the timing mode.

A. Synchronization between the two fractional hyperchaotic systems

In the general case, synchronization of chaotic fractional-order systems, united has the form:

$$\frac{d^\alpha X}{dt^\alpha} = f(X), \frac{d^\alpha Y}{dt^\alpha} = \hat{g}(Y) + U(t) \quad (13)$$

where α - order of derivative, $\alpha \in (0, 1]$, master's $X \in R^n$ and slave $Y \in R^n$ systems; $f : R^n \rightarrow R^n$ and $\hat{g} : R^n \rightarrow R^n$ - vector fields master and slave systems. In the general case condition synchronization systems defined as: $U(t) = (u_1, \dots, u_n)^T$, i.e. $\lim_{t \rightarrow \infty} \|X - Y\|$, where $\|\cdot\|$ - Euclidean norm.

B. Topological control

Consider the following general structure of the fractional-order nonlinear system under control:

$${}_0 D_t^\alpha x(t) = f(x(t) + Bu(t)), \quad (14)$$

where $u(t) = [u_1(t) u_2(t) \dots u_m(t)]^T$ is m - dimensional input vector that will be used and following control structure will be considered for state feedback:

$$u(t) = u_{eq}(t) + u_{sw}(t), \quad (15)$$

where $u_{eq}(t)$ is equivalent control and $u_{sw}(t)$ is the switching control of the system (10).

With regard to the task the topological control will be submitting a number of definitions [11].

Definition 4. The system topologically controllable if and only if coincides with \hat{x} on the basis of the criterion metrics “proximity” Hausdorff.

Theorem 2 [12]. Let E and F is compact subset R^n , $\varepsilon > 0$. Hausdorff distance $H(E, F)$ satisfies the relation.

$$H(E, F) \leq \hat{\varepsilon} \Leftrightarrow E \subset F + \hat{\varepsilon} \text{ and } F \subset E + \hat{\varepsilon},$$

where $\hat{\varepsilon} > 0$ the allowable threshold.

Topological control of the formation of a new structure made by the algorithm – “a comparison of the prototype”.

Common challenges in the implementation of topological synchronization and topological control is a base – definition, theorem and intelligent iterative algorithm.

Fractional-order iterative learning control scheme is given as [9,12]:

$$U_{k+p}^{(\alpha)}(t) = F(U_k(t), e_k(t)),$$

where

$$e_k(t) = Y_d(t) - Y_k(t).$$

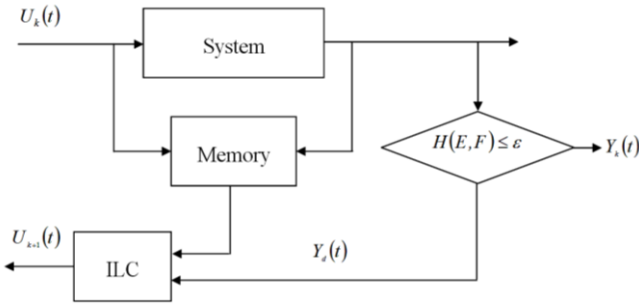


Fig. 1. The basic scheme of iterative learning control with $Y_d(t)$ being the trajectory, $U_k(t)$ and $Y_k(t)$ the input signal.

Definition 4. Two systems topologically controllable if and only if they are synchronized topologically.

Remark. If you synchronize at the same time a regularity at the exit system there is a situation called *passive control*.

Otherwise the usual iterative procedure for the organization of regular structure on the system output.

C. Formation of loss memory

The [9] mentioned that Poincare return time for chaotic systems are determined by the well-known fractal dimension of the process. The spectrum of the Poincare return time on the reversibility is chaotic systems as well hence the equivalence between spectrum and distribution generalized memory.

Therefore, memory loss is determined by the difference between the global and local fractal dimensions.

Thus required to resolve the problem in the context of the generalized memory.

D. Poincare recurrence diagram

Displayed the system on the two-dimensional square matrices $[N, N]$ and of formula [13]:

$$R_{i,j}^{m,\varepsilon} = \theta(\varepsilon_i - \|x_i - x_j\|), \quad i, j = 1, \dots, N, \quad i \neq j, \quad x \in U,$$

where N - number of considered (examined) condition x_i ;

ε - size of a neighborhood of a point x at the moment i ;

$\|\cdot\|$ - norm; $\theta(\cdot)$ - function of Heaviside.

E. Topological stability of hyperchaotic - order systems

Determine the stability of the zero solution on the system

$$\frac{dx}{dt} = {}_q \hat{\Omega}, \quad {}_q \hat{\Omega} = \{\omega_n\}_{n=0}^N, \quad {}_q \hat{\Omega} \in {}_q \hat{\Psi}.$$

Proposition. Let $GM \in U$ be structure of generalized memory. If there exists a differentiable observed $V: R^n \rightarrow R^+$ such that the following hold:

(i) - if trajectory will pass thought the point 0, i.e.

$\dot{V}(x) \leq 0$, the system is stable with $d_f(\langle \tau \rangle)$ and matches GM ;

(ii) - if trajectory will pass below the point 0, the

system is asymptotically stable with $d_f(\langle \tau \rangle)$.

IV. MAIN THEORETICAL RESULTS

A. Mathematic model of T -synchronization, tracing control and stability with generalized memory (GM).

Let mathematical model of the network of fractional order in the structure “topological synchronization - tracing control - stability” of the form

$$D^q x_i = S(x_i) + C(x_i) + \hat{S}(x_i) + \eta(x_i) + GM(x_i) + \aleph(x_i) \quad (16)$$

where $S(x_i)$ - synchronization of algorithm i -th systems, $i = 1, 2, \dots, N$;

$C(x_i)$ - control algorithm i -th systems,

$i = 1, 2, \dots, N$;

$\hat{S}(x_i)$ - stability i -th systems, $i = 1, 2, \dots, N$;

$\eta: D^q x_i \vee A \text{ frac} \left(\frac{x}{T} + \varphi \right)$, where $\text{frac}(x)$ is

the fractional part of the sawtooth;

$GM(x_i)$ - Generalized systems whit memory;

$\aleph: D^q x_i \Rightarrow PL$, PL - percolation lattice.

Let $U \in (X, \langle \tau \rangle)$ is tensioned the mathematical of the system $D^q(x_i)$, i.e.

$$D^q(x_i) \rightarrow U. \quad (17)$$

So the analyses and synthesis of model (9) components will be implemented in context of compact metric Poincare space with dimension $\langle \tau \rangle$.

In this section consider the structure “topological synchronization - tracing control - stability” of the resolution on the example of fractional order hyperchaotic systems.

B. Information processes in the kinetic model

In the fractional kinetic model of the network an important place occupied by the problem of transport information.

In this context arbitrary kinetic structure with elements of heterogeneous fractional-order chaotic maps embedding on percolation lattice that is carried out the operation in order to from structure of relations links.

Remark. The formal foundation of system embedding is the results of modern general algebra on embedding algebraic structures in relative simple algebraic structures.

Definition 5. Embedding kinetic model in n -dimensional lattice is a map in which the vertices (of the system) are map in the lattice nodes and edges (connections) are on the lattice lines.

Axiom embedding. Let $G = (V, E')$ - kinetic structure, where V - set of vertices, $V = \{g_i\}$, E' - set of edges $E' = \{e_i\}$, $i = 1, 2, \dots, n$.

Let the set of n -dimensional lattice $L \times L$ in E^2 with many of nodes, $M = \{m_j\}$ and many links, $C = \{c_j\}$, $j = 1, 2, \dots, k$.

Let some operator τ embedding $G = (V, E')$ in n -dimensional lattice $Z = (M, C)$ has the form [14]:

$$\tau : G \Rightarrow Z, \quad (18)$$

where

$$G \Rightarrow Z = \begin{cases} 1, & V \rightarrow M \\ & E' \rightarrow C \\ 0, & \text{otherwise.} \end{cases} \quad (19)$$

Then the percolation lattice will be a geometric and dynamic implementation of fractional-order kinetic model of the network.

To describe the topology of a set used geometric characteristics of the fractional – called *index of connectivity* [15].

It is known that the fractal dimension of percolation cluster in general is defined as [3]:

$$D = d - \frac{\beta}{\gamma}, \quad (20)$$

where d - space of a two dimensional flow in E^2 ; β - coefficient for two-dimensional percolation; $\gamma = 5/36 = 0.1389$; $\gamma = 1/2$ - index of correlation.

However, the real fractal dimension d_f is determined by the embedding of heterogeneous systems in a percolation lattice.

Remark. In principle, in general, there should be similarity of these dimensions, i.e.

$$D \stackrel{\text{def}}{\sim} d_f. \quad (21)$$

It is important to note that the cluster will determine it's the throughput, i.e.

$$\rho : \mu \rightarrow d_f, \quad (22)$$

where μ - the throughput; ρ - operator.

Thus, in the article the task of implementing “topological synchronization - tracing control - stability” structure fractional percolation lattice of the cases the presence without loss of memory and the memory loss.

C. Algorithm

Step 1. Let given to the fractional-order hyperchaotic system [16]:

$$\begin{aligned} D_{x_1}^q &= a(y_1 - x_1) + w_1, \\ D_{y_1}^q &= bx_1 + cx_1z_1 + w_1, \\ D_{z_1}^q &= -e^{x_1y_1} - dz_1, \\ D_{w_1}^q &= hy_1. \end{aligned} \quad (23)$$

Here, $a = 10$, $b = 40$, $c = 2$, $d = 2.5$, $h = -4$, where q is the fractional-order, $q = 0.95$.

Fractional-order hyperchaotic Chen system following [17]:

$$\begin{aligned}
 D_{x_1}^{q_1} &= a_1(x_2 - x_1) + x_4, \\
 D_{x_2}^{q_2} &= \gamma x_1 - x_1 x_3 + c_1 x_2, \\
 D_{x_3}^{q_3} &= x_1 x_2 - b_1 x_3, \\
 D_{x_4}^{q_4} &= x_2 x_3 + d_1 x_4,
 \end{aligned}
 \tag{24}$$

where $q_1 = q_2 = q_3 = q_4 = 0.95$, the system parameters are

$$(a_1, b_1, c_1, d_1, \gamma) = (35, 3, 28, 7).$$

Step 2. Simulation of the system (23) - (24) according to the algorithm [9].

Step 3. Let $\hat{x} = \{x_n\}_{n=0}^N$ is related observable two fractional-order hyperchaotic (23) and (24) systems.

Step 4. In order to achieve the behavior of synchronization between two new hyperchaotic systems by using the proposed method, suppose the master is \hat{x} .

Step 5. The related observable \hat{x} perturb of sawtooth wave $\eta: D^q x_i \vee A \text{frac}\left(\frac{x}{T} + \varphi\right)$, where $\text{frac}(x)$ is the fractional part.

$\text{frac}(x) = x - [x]$, A is amplitude, T is the period of the wave, and φ is its phase.

$\hat{Y} = \hat{x} + S(x)$, $\hat{Y} = \{y_n\}_{n=0}^N$, $\hat{Y} \in U$. \hat{Y} is slave system (fig.1).

Step 6. Iterative learning algorithm for topological synchronization on schematic “master-slave” with while tracking control for hyperchaotic nonlinear fractional-order systems shows figures 2- *a, b, c, d, e, f, g*.

Step 7. On the basis theorem and on iterative procedure define the *effect* of the “proximity” capture of the average Poincare return time as a criterion for the hyperchaotic topologically synchronization systems while tracking control.

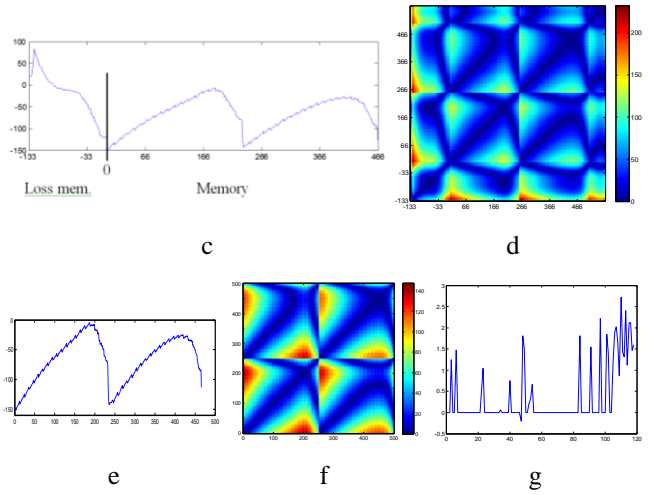


Fig.2 Visualization of iterative algorithm
a,b – first iteration: memory allocation and its recurrence plot;
c,d – next iteration: memory allocation and its recurrence plot;
e,f – real memory and its recurrence plot; g – Lyapunov indicator of chaoticity, $\text{lyap.min}=-0,2009$, $\text{lyap.max}=2,7177$.

Step 8. Define the Poincare diagram $D\hat{x}, D\hat{y}$ [13] and fractal dimensions:

$$d_{f_1} = 1.2586, \quad d_{f_2} = 1.2034;$$

the average Poincare return time as:

$$\langle \tau \rangle_1 = 3.9721428, \quad \langle \tau \rangle_2 = 3.798857.$$

Step 9. Effects of the “proximity” define as:

$$\begin{aligned}
 J(\langle \tau \rangle_1, \langle \tau \rangle_2) \leq \hat{\epsilon} &\Leftrightarrow \langle \tau \rangle_1 \overset{\text{prox}}{\subset} \langle \tau \rangle_2 + \hat{\epsilon}, \\
 \langle \tau \rangle_2 \overset{\text{prox}}{\subset} \langle \tau \rangle_1 + \hat{\epsilon}, &\text{ where } \hat{\epsilon} = 0.17.
 \end{aligned}$$

The figure 1, *f* shows regularity-like of tracking control.

Step 10. Define Lyapunov stability.

i) Let the Lyapunov function is given in the quadratic form as:

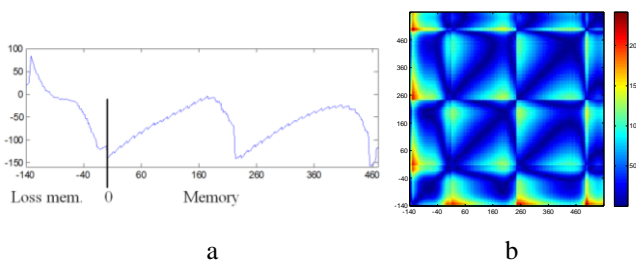
$$V(x) = \frac{1}{2^q} \Omega^2.$$

ii) We calculate the total derivative of the function

$V(x)$:

$$\frac{dV}{dt} = \frac{\partial V}{\partial x} \frac{dx}{dt} = {}_q \Omega_q \hat{\Omega} \leq 0,$$

where $\hat{\Omega} \rightarrow$ memory function (fig. 3).



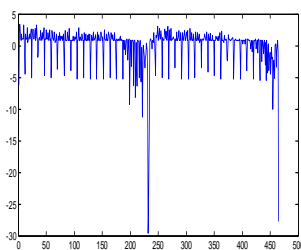


Fig.3. Fractional Lyapunov stability.

Step 11. We make embedding fractional kinetic models in percolation lattice of rules (18) and (19). On fractional percolation lattice is a series connection of heterogeneous hyperchaotic fractional mapping (fig.4.).



Fig.4. Structured fractional kinetic model.

The size of the lattice 16×16 . Share indignation number: 0.763. Number of runs the program: 50. The probability of membership node in percolation cluster: 0.95249. The average cluster size – 4.6076. Probability emergence percolation cluster – 0.96.

V. CONCLUSION

Resolution structure “topological synchronization – control – stability” showed method analyses from generalized memory. When tracing control process formed the transition of “hyperchaos-chaos” confirmed by a positive Lyapunov exponent.

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