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Structure Quasi-Po-Ternary Ideals in Po-Ternary Semirings

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Abstract— **In this paper we introduce the definitions related to quasi-PO-ternary ideals and Bi-PO-ternary ideals in PO-ternary semirings and we study the relation between quasi-PO-ternary ideals and Bi-PO-ternary ideals in PO-ternary semirings.**

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Index Terms — Quasi PO-k-ternary ideal, quasi-PO-ternary ideal of T generated by A, quasi simple, 0-quasi simple, quasi k-PO-ternary ideal.

I. Introduction

The notion of semiring was introduced by Vandiver [III] in 19III4. In fact semiring is a generalization of ring. In 1971 Lister [2] characterized those additive subgroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. MadusudhanaRao. D, Siva Prasad. P and SrinivasaRao. G [4, 5, 6, 7, 8] studied and investigated some results on partially ordered ternary semiring.

II. PRELIMINARIES

Definition II.1[6] : A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semiring*** if T is an additive commutative semigroup satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,
ii) $[(a + b)cd] = [acd] + [bcd]$,
iii) $[a(b + c)d] = [abd] + [acd]$,
iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in T$.

Note II.2[6] : For the convenience we write $x_1x_2x_3$ instead of $[x_1x_2x_3]$

Note II.3[6]: Let Tbe a ternary semiring. If A, B and C are three subsets of T, we shall denote the set ABC = $\{\Sigma abc : a \in A, b \in B, c \in C\}$.

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Note II.4[6] : Let T be a ternary semiring. If A, B are two subsets of T, we shall denote the set A + B = $\{a+b : a \in A, b \in B\}$ and 2A = { $a + a$: $a \in$ A}.

Note II.5[6] : Any semiring can be reduced to a ternary semiring.

Definition II.6 [6]: A ternary semiring T is said to be a ***partially ordered ternary semiring*** or simply ***PO Ternary Semiring*** or ***Ordered Ternary Semiring*** provided T is partially ordered set such that $a \leq b$ then

(1) $a + c \leq b + c$ and $c + a \leq c + b$,
(2) $acd \leq bcd$, $cad \leq cbd$ and $cda \leq cdb$ for all $a, b, c, d \in$ T.

Throughout T will denote as PO-ternary semiring unless otherwise stated.

Theorem II.7 [6]: Let Tbe a po-ternary semiring and A ⊆T, B ⊆T and C ⊆ T. Then (i) A ⊆(A], (ii) ((A]] = (A], (iii) (A](B](C] ⊆(ABC] and (iv) A ⊆B ⇒A ⊆(B] and (v) A ⊆B ⇒(A] ⊆(B], (vi) (A ∩ B] = (A] ∩ (B], (vii) (A ∪ B] = (A] ∪ (B].

Definition II.8 [6]: A nonempty subset A of a PO-ternary semiring T is a***PO-ternary ideal*** of T provided A is additive subsemigroup of T, ATT ⊆ A, TTA ⊆ A, TAT ⊆ A and (A] ⊆ A.

Theorem II.9[8] : Let T be a PO-ternary semiring and A, B be two PO-ternary ideals of T, then the sum of A, B denoted by A + B is a PO-ternary ideal of T where A + B = {*x* = *a* + *b* / *a*∈ A, *b*∈ B}.

Theorem II.10[8]: Let A be a PO-ternary ideal of T. Then (A] is an ordered ternary ideal of T generated by A.

Theorem II.11[8] : The left PO-ternary ideal of a PO-ternary semiring T generated by a non-empty subset A is the intersection of all left PO-ideals of T containing A.

Theorem II.12[8] : The lateral ideal of a ternary semiring T generated by a non-empty subset A is the intersection of all lateral ideals of T containing A.

Theorem II,13[8] : The right PO-ternary ideal of a PO-ternary semiring T generated by a nonempty subset A is the intersection of all right PO-ternary ideals of T containing A.

III. QUASI-PO-TERNARY IDEALS

We now introduce the notion of quasi-PO-ternary ideals in PO-ternary semirings.

Definition III.1: A non-empty subset Q of a PO-ternary semiring T is said to be ***quasi-PO-ternary ideal***provided Q is a subsemigroups of (T, +) satisfying
(1) TTQ ∩ TQT ∩ QTT ⊆ Q
(2) TTQ∩ TTQTT∩ QTT⊆ Q and
(III) (Q] ⊆ Q.

Example III.2: Let T = $M_2(Z_0^-)$ be the PO-ternary semiring of the set of all 2 × 2 square matrices over Z_0^-, the set of all non positive integers. Let Q = $\left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in Z_0^-\right\}$. Then we can easily verify that Q is a quasi-PO-ternary ideal of T.

Theorem III.3: Every left PO-ternary ideal of a PO-ternary semiring T is a quasi-PO-ternary ideal of T.
Proof: Assume that Q is a left PO-ternary ideal of T. Then (TTQ]⊆ Q, but (TTQ] ∩ (TQT ∪ TTQTT] ∩ (QTT] ⊆(TTQ]⊆ Q. Hence Q is a quasi-PO-ternary ideal of T.

Theorem III.4:Every lateral PO-ternary ideal of a PO-ternary semiring T is a quasi-PO-ternary ideal of T.
Proof :Similar to III.III.

Theorem III.5:Every right PO-ternary ideal of a PO-ternary semiring T is a quasi-PO-ternary ideal of T.
Proof :Similar to III.III.

Theorem III.6:Every two sided PO-ternary ideal of a PO-ternary semiring T is a quasi-PO-ternary ideal of T.
Proof : Any two sided PO-ternary ideal of T is a left PO-ternary ideal and right PO-ternary ideal and any left PO-ternary or any right PO-ternary ideal of T is a quasi-PO-ternary ideal of T. Therefore any two sided PO-ternary ideal of T is a quasi-PO-ternary ideal of T.

Theorem III.7:Every PO-ternary ideal of a PO-ternary semiring T is a quasi-PO-ternary ideal of T.
Proof :Similar to III.6.

Note III.8: In general a quasi-PO-ternary ideal need not be a left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T.

Example III.9: In example III.2, Q is a quasi-PO-ternary ideal of T, but Q is not left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T.

Theorem III.10: Let T be a commutative PO-ternary semiring, then every quasi-PO-ternary ideal of T is a three sided PO-ternary ideal of T.
Proof: Assume that T be a commutative PO-ternary semiring. Let Q be a quasi-PO-ternary ideal of T. Then (TTQ]∩ (TQT ∪ TTQTT]∩(QTT]⊆ Q. Since T be a commutative and Q ⊆ T, then TTQ = TQT = QTT = TTQTT. Now TTQ ∩ (TQT ∪ TTQTT) ∩ QTT = (TTQ ∩ TQT ∩ QTT) ∪ (TTQ ∩ TTQTT ∩ QTT) ⊆TTQ ∪TTQ =(TTQ] + (TTQ] = (TTQ]⊆ Q. Hence Q is a left PO-ternary ideal of T. Similarly Q is lateral PO-ternary ideal and right PO-ternary ideal of T. Therefore, every quasi-PO-ternary ideal of T is a three sided ideal of T.

Theorem III.11: The intersection of any system of quasi-PO-ternary ideals is a quasi-PO-ternary ideal of T or empty.
Proof: Let $\{Q_\alpha\}_{\alpha\in\Delta}$ be a family of PO-ternary ideals of T and let Q = $\bigcap_{\alpha\in\Delta} Q_\alpha$

Assume that Q is not empty. Since Q_α is a quasi-PO-ternary ideal for each $\alpha \in \Delta$. Then $(Q_\alpha TT] \cap (TQ_\alpha T + TTQ_\alpha TT] \cap (TTQ_\alpha] \subseteq Q_\alpha$ for each $\alpha \in \Delta$.

Now for each $\alpha\in\Delta$ TTQ = TT($\bigcap_{\alpha\in\Delta} Q_\alpha$) = $\bigcap_{\alpha\in\Delta} TTQ_\alpha \subseteq$ $TTQ_\alpha \subseteq (TTQ_\alpha]$, TQT= T($\bigcap_{\alpha\in\Delta} Q_\alpha$)T = $\bigcap_{\alpha\in\Delta} TQ_\alpha T \subseteq$ $TQ_\alpha T \subseteq (TQ_\alpha T]$, TTQTT = TT($\bigcap_{\alpha\in\Delta} Q_\alpha$)TT = $\bigcap_{\alpha\in\Delta} TTQ_\alpha TT \subseteq TTQ_\alpha TT \subseteq (TTQ_\alpha TT]$, and TTQ = TT($\bigcap_{\alpha\in\Delta} Q_\alpha$) = $\bigcap_{\alpha\in\Delta} Q_\alpha TT \subseteq Q_\alpha TT \subseteq (Q_\alpha TT]$. Then (TTQ]∩ (TQT ∪ TTQTT]∩(QTT]⊆$(TTQ_\alpha]\cap(TQ_\alpha T \cup TTQ_\alpha TT]\cap(Q_\alpha TT] \subseteq Q_\alpha$ for each $\alpha\in\Delta$. Therefore (TTQ] ∩ (TQT + TTQTT] ∩ (QTT]⊆ $\bigcap_{\alpha\in\Delta} Q_\alpha$ = Q.

Let $x \in \bigcap_{\alpha\in\Delta} Q_\alpha$ and $y \in$ T be such that $y \le x$. Let for each $\alpha\in\Delta$. Since $y \le x$ and $x \in Q_\alpha$, $y\in Q_\alpha$. Thus $y \in \bigcap_{\alpha\in\Delta} Q_\alpha$ (i.e. $(Q] = \left(\bigcap_{\alpha\in\Delta} Q_\alpha\right] \subseteq \bigcap_{\alpha\in\Delta} (Q_\alpha] = \bigcap_{\alpha\in\Delta} Q_\alpha = Q$). Therefore Q = $\bigcap_{\alpha\in\Delta} Q_\alpha$, is a quasi-PO-ternary ideal of T.

Theorem III.12: Every quasi-PO-ternary ideal of a PO-ternary semiring T is a PO-ternary subsemiring of T.
Proof :Let Q be a quasi-PO-ternary ideal of T. By definition III.1, Q is a subsemigroup of (T, +). Let $a, b, c \in Q \subseteq T$. Then abc∈(TTQ], abc∈(TQT], and abc∈(QTT]. Therefore abc∈(TTQ]∩ (TQT ∪ TTQTT]∩(QTT]⊆ Q, since Q is a quasi-PO-ternary ideal of T and hence abc∈ Q. Therefore Q is a PO-ternary subsemiring of T.

Lemma III.13: If Q is a quasi-PO-ternary ideal of a PO-ternary semiringT and S is a PO-ternary subsemiringof T, then Q∩S is a quasi-ideal of S.
Proof: Assume that Q_1 = Q∩S ≠ ∅. Since Q_1⊆ Q, it follows that $SSQ_1 \cap SQ_1S \cap Q_1SS \subseteq TTQ \cap TQT \cap QTT \subseteq Q$. Since Q_1⊆ S and S is a PO-ternary subsmigroup of T. We have $SSQ_1 \cap SQ_1S \cap Q_1SS \subseteq S$. Then $SSQ_1 \cap SQ_1S \cap Q_1SS \subseteq Q_1$.Let $x\in Q_1$ and y∈ S such that $y \le x$. Since x∈ Q, $y \in$ Q.So $y \in Q_1$. Therefore Q_1 is quasi-PO-ternary ideal of S.

Theorem III.14: The intersection of left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal is a quasi-PO-ternary ideal of T.

Proof **:**Let L, M, and R be the left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T respectively. Let Q = L ∩ M ∩ R. Choose l∈ L, m∈ M, r∈ R. Since lmr∈ L ∩ M ∩ R, Q is not empty. Since TTQ ⊆ L, TQT ⊆ M and QTT ⊆ R, then we have TTQ ∩ TQT ∩ QTT ⊆ L ∩ M ∩ R = Q. Similarly, TTQ ∩ TTQTT ∩ QTT ⊆ Q.Let x∈ L ∩ M ∩ R and y∈ T such that $y \le x$. Since x∈L ∩ M ∩ R, y∈L ∩ M ∩ R. Therefore Q = L ∩ M ∩ R is a quasi-PO-ternary ideal of T.

Theorem III.15: An additive subsemigroup Q of PO-ternary semiring T is a quasi-PO-ternary ideal of T, if Q is the intersection of a left PO-ternary ideal, a lateral PO-ternary ideal and a right PO-ternary ideal of T.

Proof **:**Assume that Q is a quasi-PO-ternary ideal of T and L = (TTQ ∪ Q], M = (TQT ∪ TTQTT ∪ Q], R = (QTT ∪ Q], then by theorems 2.10, III.4, III.5, III.6, we have L is left PO-ternary ideal, M is lateral PO-ternary ideal and R is right PO-ternary ideal of T containing Q respectively. Thus Q ⊆ L ∩ M ∩ R. Since Q is quasi-PO-ternary ideal of T. We have

L ∩ M ∩ R = (TTQ ∪ Q] ∩(TQT ∪ TTQTT ∪ Q] ∩ (QTT ∪ Q] = ((TTQ] ∩ (TQT ∪ TTQTT] ∩ (QTT]) ∪ (Q] ⊆ Q ∪ (Q] = Q.. Therefore Q = L ∩ M ∩ R and hence Q is the intersection of left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T.

Definition III.16: Let A be a nonempty subset of an ordered ternary semi-ring T. The intersection of all quasi-PO-ternary ideals of T containing A is calledthe ***quasi-PO-ternary ideal of T generated by A*** and is denoted by Q(A). Moreover,Q(A) is the smallest quasi-PO-ternary ideal of T containing A. If A = {a}, wealso write Q({a}) as Q(a) or <a>$_q$.

Theorem III.17:Let A be a nonempty subset of an ordered ternary semi-ring T. Then
Q(A) = (A]∪ [((TTA]∩(TAT∪TTATT]∩(ATT]).
In particular, Q(a) = <a>$_q$ = (a] ∪((TTa] ∩(TaT∪TTaTT] ∩(aTT]) for all a ∈T.

Proof: By the theorem II.11, II.12, and II.13, we have (A ∪ TTA], (A ∪ TAT ∪ TTATT] and (A ∪ ATT] are left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T containing A, respectively. By theorem III.15, we have (TTA ∪ A] ∩ (TAT ∪ TTATT ∪ A] ∩ (ATT ∪ A] is a quasi-PO-ternary ideal of T containing A. Thus Q(A) ⊆ (TTA ∪ A] ∩ (TAT ∪ TTATT ∪ A] ∩ (ATT ∪ A]

= (A] ∪ ((TTA] ∪ (TAT ∪ TTATT] ∪ (ATT]). By the theorem III.15, we have

(A] ∪ ((TTA] ∪ (TAT ∪ TTATT] ∪ (ATT])
= (TTA ∪ A] ∩ (TAT ∪ TTATT ∪ A] ∩ (ATT ∪ A]
⊆ (TTQ(a) ∪ Q(A)] ∩ (TQ(A)T ∪ TTQ(A)TT] ∩ (TTQ(A) ∪ Q(A)]⊆ Q(A).

Hence Q(A) = (A] ∪ ((TTA] ∪ (TAT ∪ TTATT] ∪ (ATT]).

Now we characterize the relationship between the minimality of the quasi-PO-ternary ideals and a quasi-simple and 0-quasi simple-PO-ternary semirings.

Definition III.18 : Let T be a PO-ternary semiring with a zero element. Then T is called ***quasi simple*** if T has no proper quasi-PO-ternary ideals of T.

Theorem III.19: Let T be a PO-ternary semiring without a zero element. Then the following are equivalent.
(1) T is a quasi-simple.
(2) (TTa] ∩ (TaT∪TTaTT] ∩ (aTT] = T for all a∈ T.
(III) Q(a) = T for all a∈ T.

Proof: (1) ⇒ (2) :Suppose that T is quasi-simple and let a∈T. By the theorem 2.11, 2.12, and 2.1III, we have (a∪TTa], (a∪TaT∪TTaTT] and (a∪aTT] are left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T containing A, respectively. By theorem III.15, we have (TTa∪a] ∩ (TaT∪TTaTT∪a] ∩ (aTT∪a] is a quasi-PO-ternary ideal of T containing a. Since T is quasi-simple, thus (TTa∪a] ∩ (TaT∪TTaTT∪a] ∩ (aTT∪a] = T.

(2) ⇒ (III) :Assume that (TTa] ∩ (TaT∪TTaTT] ∩ (aTT] = T for all a∈ T**.** By theorem III.18, we have T = (TTa] ∩ (TaT∪TTaTT] ∩ (aTT]) ⊆(a] ∪ ((TTa] ∩ (TaT∪TTaTT] ∩ (aTT]) = Q(a). Therefore Q(a) = T.

(III) ⇒ (1) :Assume that Q(a) = T for all a∈ T**.** Let Q be a quasi-PO-ternary ideal of T and let a∈ Q**.** Then Q(a) = T, and so Q(a) ⊆ Q ⊆ T. Therefore T = Q and hence T is quasi-simple.

Definition III.20: Let T be a PO-ternary semiring with zero element, T^{III}≠ {0} and | T | > 1. Then T is called ***0-quasi-simple***if T has no non zero proper quasi-PO-ternary ideals.

Theorem III.21: Let T be a PO-ternary semiring with zero element, T^{III}≠ {0} and | T | > 1. Then T is a 0-quasi-simple if and only if Q(a) = T for a∈ T\{0}.

Proof: Suppose that T is a 0-quasi-simple and a∈ T\{0}. Then Q(a) ≠ {0}. Since T is 0-quasi simple, therefore Q(a) = T.

Conversely, suppose that Q(a) = T for all a∈ T\{0}. Let Q be a non-zero quasi-PO-ternary ideal of T and a∈ Q\{0}. Then Q(a) = T and Q(a) ⊆ Q ⊆ T implies that T = Q. Therefore, T is a 0-quasi-simple.

Definition III.22: Aleft quasi-PO-ternary ideal Q of an ordered ternary semiringT without a zero element is called a ***minimal left quasi-PO-ternary ideal*** of T if thereis no aleft quasi-PO-ternary ideal A of T such that A ⊆Q. Equivalently, if for anyleft quasi-PO-ternary ideal A of T such that A ⊆Q, we have A = Q.

Definition III.23: Alateral quasi-PO-ternary ideal Q of an ordered ternary semiringT without a zero element is called a ***minimal lateral quasi-PO-ternary ideal*** of T if thereis no alateral quasi-PO-ternary ideal A of T such that A ⊆Q. Equivalently, if for any lateral quasi-PO-ternary ideal A of T such that A ⊆Q, we have A = Q.

Definition III.24: Aright quasi-PO-ternary ideal Q of an ordered ternary semiringT without a zero element is called a ***minimal right quasi-PO-ternary ideal*** of T if thereis no aright quasi-PO-ternary ideal A of T such that A ⊆Q. Equivalently, if for any right quasi-PO-ternary ideal A of T such that A ⊆Q, we have A = Q.

Definition III.25: Atwo sided quasi-PO-ternary ideal Q of an ordered ternary semiringT without a zero element is called a ***minimal two sided quasi-PO-ternary ideal*** of T if thereis no atwo sided quasi-PO-ternary ideal A of T such that A ⊆Q.

Equivalently, if for any two sided quasi-PO-ternary ideal A of T such that A ⊆Q, we have A = Q.

Definition III.26: A quasi-PO-ternary ideal Q of an ordered ternary semiringT without a zero element is called a ***minimal quasi-PO-ternary ideal*** of T if thereis no a quasi-PO-ternary ideal A of T such that A ⊆Q. Equivalently, if for anyquasi-PO-ternary ideal A of T such that A ⊆Q, we have A = Q.

Theorem III.27: Let Q be a quasi-PO-ternary ideal of an ordered ternary semi-ring T without a zero element. Then Q is a minimal quasi-PO-ternary ideal of T if and only if it is the intersection of a minimal ordered left, a minimal ordered right and a minimal ordered lateral PO-ternary ideal of T.

Proof: Suppose that Q is a minimal quasi-PO-ternary ideal of T. Then(TT*Q*] ∩ (T*Q*T ∪ TT*Q*TT] ∩ (*Q*TT]) ⊆ Q. By the theorems2.11, 2.12, and 2.1III, we have (TT*Q*], (T*Q*T∪ TT*Q*TT] and (*Q*TT] are left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T and by theorem III.15, we have (TT*Q*] ∩ (T*Q*T ∪ TT*Q*TT] ∩ (*Q*TT]) is a quasi-PO-ternary ideal of T.

Since Q is minimal quasi-PO-ternary ideal of T.

We have Q = (TT*Q*] ∩ (T*Q*T ∪ TT*Q*TT] ∩ (*Q*TT]).

We claim that (TTQ] is a minimal left PO-ternary ideal of T. Let L be a left PO-ternary ideal of T such that L ⊆ (TTQ]. Then (TTL] ⊆(L] = L ⊆(TTQ].

Therefore, (TTL]∩(TQT ∪TTQTT]∩(QTT] ⊆(TTQ]∩ (TQT ∪TTQTT]∩(QTT] = Q.

Since (TTL]∩(TQT∪TTQTT]∩(QTT] is a quasi-PO-ternary ideal of T and Q is aminimal quasi-PO-ternary ideal of T,we have (TTL]∩(TQT ∪TTQTT]∩(QTT] =Q.

Thus Q ⊆(TTL] and so (TTQ] ⊆ (TT(TTL]] ⊆(TT(L]] = (TTL] ⊆L.

Hence, L = (TTQ]. Therefore, (TTQ] is a minimal left PO-ternary ideal of T.

Similarly, we can show that (QTT] and (TQT [TTQTT] areminimal right PO-ternary ideal and minimal lateral PO-ternary ideal of T, respectively.

Conversely, let Q = L ∩ M ∩ R where L, M and R are a minimal left PO-ternary ideal, a minimal lateral PO-ternary ideal and a minimal rightPO-ternary ideal of a PO-ternary semiringT, respectively. By theorem III.15, Q is a quasi-PO-ternary ideal of T. Let A be a quasi-PO-ternary ideal of T such that A ⊆ Q. By theorems 2.11, 2.12, and 2.1III, we have (TT*A*], (T*A*T ∪ TT*A*TT] and (*A*TT] are left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideal of T. Now (TTA] ⊆(TTQ] ⊆(TTL) ⊆(L] = L. Since L is a minimal left PO-ternary ideal of T, we have (TTA] = L.Similarly, (TAT ∪ TTATT] = M and (ATT] = R. Since A is a quasi-PO-ternary ideal of T and hence

Q = L ∩ M ∩ R = (TT*A*] ∩ (T*A*T ∪ TT*A*TT] ∩ (*A*TT] ⊆ A. Therefore A = Q. Hence, Q is a minimal quasi-PO-ternary ideal of PO-ternary semiringT.

Theorem III.28: Let Q be a quasi-PO-ternary ideal of an ordered ternary semi-ring T without a zero element. If Q is quasi-simple, then Q is a minimal quasi-PO-ternary ideal of T.

Proof :Suppose that Q is quasi-simple and let A be a quasi-PO-ternary ideal ofT such that A ⊆Q. Therefore (QQA] ∩ (QAQ ∪ QQAQQ] ∩ (AQQ] ⊆ (TTA] ∩ (TAT ∪ TTATT] ∩ (ATT] ⊆ A and (A] ∩ Q ⊆ (A] = A. Therefore A is a quasi-PO-ternary ideal of Q. Since Q is quasi-simple and hence Q = A. Hence Q is a minimal quasi-PO-ternary ideal of T.

Theorem III.29: Let T be an ordered ternary semiring without a zero element having proper quasi-PO-ternary ideals. Then every proper quasi-PO-ternary ideal of T is minimal if and only if the intersection of any two distinct proper quasi-PO-ternary ideals is empty.

Proof: Let Q_1 and Q_2be two distinct proper quasi-PO-ternary ideals of T. Byassumption, we have that Q_1 and Q_2 are minimal.If $Q_1 \cap Q_2 \neq \emptyset$, then by Theorem III.11, $Q_1 \cap Q_2$is a quasi-PO-ternary ideal of T.Since $Q_1 \cap Q_2 \subseteq Q_1$ andQ_1 is minimal, we have $Q_1 \cap Q_2 = Q_1$.Since $Q_1 \cap Q_2 \subseteq Q_2$ andQ_2 is minimal, we have $Q_1 \cap Q_2 = Q_2$. Therefore $Q_1 = Q_1 \cap Q_2 = Q_2$. This is a contradiction and hence $Q_1 \cap Q_2 = \emptyset$.

Conversely, suppose that Q be a proper quasi-PO-ternary ideal of T and let A be a quasi-PO-ternary ideal of T such that A ⊆Q. Then A is a proper quasi-PO-ternary idealof T. If A ≠ Q, then by assumption, A = A∩Q = ∅. That is acontradiction. Hence, A = Q. Therefore, Q is a minimal quasi-PO-ternary idealof T.

Definition III.30: A nonzero quasi-PO-ternary ideal Q of a PO-ternarysemiring T with a zero element is called a ***0-minimal quasi-PO-ternary ideal***of T if there is no a nonzero quasi-PO-ternary ideal A of T such that A ⊆Q.Equivalently, if for any nonzero quasi-PO-ternary ideal A of T such that A ⊆Q,we have A = Q.

Note III.31: We also define a 0-minimal left PO-ternary ideal, a 0-minimal lateral PO-ternary anda 0-minimal right PO-ternary ideal of an ordered ternary semiring T with a zeroelement in the same way of a 0-minimal quasi-PO-ternary ideal.

Theorem III.32: Let T be a PO-ternary semiring with a zero element. Then the intersection of a 0-minimal left PO-ternary ideal, a 0-minimal right PO-ternary ideal and a 0-minimal lateral PO-ternary ideal of T is either {0}or a 0-minimal quasi-PO-ternary ideal of T.

Proof : Let Q = L ∩ M ∩ R ≠ {0} where L, M, and R are a 0-minimal left PO-ternary ideal, a 0-minimal lateral PO-ternary ideal and a 0-minimal right PO-ternary ideal of T, respectively. By theorem III.14, Q is a quasi-PO-ternary ideal of T. Let A be a non zero quasi-PO-ternary ideal of T such that A ⊆ Q. By theorems II.11, II.12, and II.13, (TTA], (TAT ∪ TTATT], (ATT] are left PO-ternary ideal, lateral PO-ternary ideal and right PO-ternary ideals of T respectively. Then we get the following two cases:

***Case-1*:**(TTA] = {0}, (TAT ∪ TTATT] = {0}, (ATT] = {0}. If (TTA] = {0}, then (TTA] = {0} ⊆A. Thus A is a nonzero left PO-ternary ideal of T. Since A ⊆ Q ⊆ L and L is a 0-minimal left PO-ternary ideal of T. Then we have A = L. Therefore A = Q. Similarly, if (ATT] = {0} or (TAT ∪ TTATT] = {0], we get A = Q.

***Case-2*:**(TTA] ≠ {0}, (TAT ∪ TTATT] ≠ {0}, (ATT] ≠ {0}. Now (TTA] ⊆(TTQ] ⊆(TTL) ⊆(L] = L.Since L is a 0-minimal left PO-ternary ideal of T, we have (TTA] = L. Similarly,(TAT ∪TTATT] = M and (ATT] = R.

Since A is a quasi-PO-ideal ofT, we haveQ = L ∩M ∩R = (TTA] ∩(TAT ∪TTATT] ∩(ATT] ⊆A⇒ A = Q. Hence, Q is a 0-minimal ordered quasi-ideal ofT.

Theorem III.33: Let Q be anon-zero quasi-PO-ternary ideal of an ordered ternary semi-ring T with a zero element. If Q is 0-quasi-simple, then Q is a 0-minimal quasi-PO-ternary ideal of T.

Proof **:**Suppose that Q is 0-quasi-simple and let A be anon-zero quasi-PO-ternary ideal ofT such that A ⊆Q. Therefore (QQA] ∩ (QAQ ∪ QQAQQ] ∩ (AQQ] ⊆ (TTA] ∩ (TAT ∪ TTATT] ∩ (ATT] ⊆ A and (A] ∩ Q ⊆ (A] = A. Therefore A is a non-zero quasi-PO-ternary ideal of Q. Since Q is 0-quasi-simple and hence Q = A. Hence Q is a 0-minimal quasi-PO-ternary ideal of T.

Theorem III.34: Let Tbe an ordered ternary semiring with a zero element having non-zero proper quasi-PO-ternary ideals. Then every non-zero proper quasi-PO-ternary ideal of T is 0-minimal if and only if the intersection of any two distinct non-zero proper quasi-PO-ternary ideals is {0}.

Proof: Let Q_1 and Q_2be two distinct non-zero proper quasi-PO-ternary ideals of T. Byassumption, we have that Q_1 and Q_2 are 0-minimal.If $Q_1 \cap Q_2 \neq \{0\}$, then by Theorem III.11, $Q_1 \cap Q_2$is anon-zero quasi-PO-ternary ideal of T.Since $Q_1 \cap Q_2 \subseteq Q_1$ andQ_1 is 0-minimal, we have $Q_1 \cap Q_2 = Q_1$.Since $Q_1 \cap Q_2 \subseteq Q_2$ andQ_2 is 0-minimal, we have $Q_1 \cap Q_2 = Q_2$. Therefore $Q_1 = Q_1 \cap Q_2 = Q_2$. This is a contradiction and hence $Q_1 \cap Q_2 = \{0\}$.

Conversely, suppose that Q be anon-zero proper quasi-PO-ternary ideal of T and let A be anon-zero quasi-PO-ternary ideal of T such that A ⊆Q. Then A is a non-zero properquasi-PO-ternary idealof T. If A ≠ Q, then by assumption, A = A∩Q = {0}. That is acontradiction. Hence, A = Q. Therefore, Q is a 0-minimal quasi-PO-ternary idealof T.

Theorem III.35: Let *x* be an idempotent element of a PO-ternary semiringT, that is, $x^{III}(=xxx) \geq x$. If R is a right PO-ternary ideal, M a lateral PO-ternary ideal, and L a left PO-ternary ideal of T, then (R*xx*], (*xx*M*xx*],and (*xx*L] are quasi-PO-ternary ideals of T.

Proof: To show (*Rxx*], (*xxMxx*], and (*xxL*] are quasi-ideals of *S*, it is sufficient to show that

(*Rxx*] = (*R*]∩(*TxT*+*TTxTT*] ∩(*TTx*],(*xxMxx*] = (*xTT*]∩(*M*]∩(*SSx*], and

(*xxL*]= (*xTT*]∩(*TxT*∪*TTxTT*]∩(*L*].

For the first case, it is clear that (*Rxx*]⊆*R* ∩ *TTx* = (R ∩ TT*x*] = (R] ∩ (TT*x*].

Let *a*∈ (R] ∩ (TT*x*] ⇒*a*∈ (R] and *a*∈ (TT*x*].

Now, *a*∈ (TT*x*]⇒$a \leq \sum_{i=1}^{n} s_i t_i x$ for some s_i, t_i∈ T.

Therefore $axx \leq (\sum_{i=1}^{n} s_i t_i x)xx = \sum_{i=1}^{n} s_i t_i (x\,xx) \geq \sum_{i=1}^{n} s_i t_i x \geq a$.

It follows that *a*∈ (R*xx*] and hence (R*xx*] = (R] ∩ (TT*x*].

Again $a \leq axx \Rightarrow a \in$ (T*x*T]. Therefore we have *a*∈ (T*x*T∪TT*x*TT].

Thus (R] ∩ (TT*x*] ⊆ (T*x*T∪TT*x*TT]. Therefore, (*Rxx*]= (*R*]∩(*TxT*+*TTxTT*] ∩(*TTx*].

For the second case, we see that (*xxMxx*] ⊆(*xTT*] ∩ (M] ∩ (TT*x*]. Let *a*∈ (*xTT*] ∩ (M] ∩ (TT*x*]. Then *a*∈ (*x*TT], *a*∈ (M] and *a*∈ (TT*x*]. Now *a*∈ (*x*TT] and *a*∈ (TT*x*]

$\Rightarrow a \leq \sum_{i=1}^{n} s_i t_i x = \sum_{j=1}^{m} x u_j v_j$ for some s_i, t_i, u_j, v_j∈ T.

Therefore $xxaxx \leq xx(\sum_{i=1}^{n} s_i t_i x)xx$

$= xx\sum_{i=1}^{n} s_i t_i (x\,xx) \geq xx\sum_{i=1}^{n} s_i t_i x = xx\sum_{j=1}^{m} x u_j v_j =$

$\sum_{j=1}^{m} (xxx) u_j v_j \geq \sum_{j=1}^{m} x u_j v_j \geq a$

⇒ *a* ∈ (*xxMxx*] and hence (*xxMxx*]= (*xTT*]∩(*M*]∩(*SSx*].

For the third case it is similar to first case.

IV. PRIME QUASI-PO-TERNARY IDEALS

In this section, we introduce the notions of prime and semiprime quasi-PO-ternary ideals in PO-ternary semirings and some relevant counter examples are also indicated.

Definition IV.1: A proper quasi-PO-ternary ideal Q of a PO-ternary semiring T is said to be ***prime quasi-PO-ternary ideal***provided ABC ⊆ Q implies that A ⊆ Q or B ⊆ Q or C ⊆ Q for some quasi-PO-ternary ideals A, B, C of T.

Definition IV.2: A proper quasi-PO-ternary ideal Q of a PO-ternary semiring T is said to be ***semiprime quasi-PO-ternary ideal***provided A^3⊆ Q implies that A ⊆ Q for some quasi-PO-ternary ideal A of T.

Definition IV.3: A proper quasi-PO-ternary ideal Q of a PO-ternary semiring T is said to be ***weakly prime quasi-PO-ternary ideal*** provided Q ⊆ A, B ⊆ Q, C ⊆ Q and ABC ⊆ Q implies that A = Q or B = Q or C = Q for some quasi-PO-ternary ideals of T.

Theorem IV.4: Every prime quasi-PO-ternary ideal of T is a Semiprime quasi-PO-ternary ideal of T.

Proof: Suppose that Q is a prime quasi-PO-ternary ideal of T and A be any quasi-PO-ternary ideal of T such that A^3 = AAA ⊆ Q. Since Q is prime. Therefore A ⊆ Q and hence Q is a Semiprime quasi-PO-ternary ideal of T.

Note IV.5: The converse of the theorem 4.4. need not be true. i.e., every Semiprime quasi-PO-ternary ideal of T need not be a prime quasi-PO-ternary ideal of T.

Example IV.6: Let T = $M_2(Z_0^-)$ is a PO-ternary semiring of 2 × 2 square matrices over Z_0^-. Let Q = $\left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in Z_0^-\right\}$ and $a \leq b$ for $a, b \in Z_0^-$. Then Q is a Semiprime quasi-PO-ternary ideal of T. But Q is not a prime quasi-PO-ternary ideal of T.

Since A = $\left\{\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} : b \in Z_0^-\right\}$,

B = $\left\{\begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} : c \in Z_0^-\right\}$ and

C = $\left\{ \begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix} : d \in Z_0^- \right\}$ are quasi-PO-ternary ideal of T such that ABC ⊆ Q. But A ⊈ Q, B ⊈ Q and C ⊈ Q.

Theorem IV.7: Every prime quasi-PO-ternary ideal Q of a PO-ternary semiring T is a weakly prime quasi-PO-ternary ideal of T.

Proof: Suppose that Q is a prime quasi-PO-ternary ideal of T. Then there exist quasi-PO-ternary ideals A, B, C of T such that ABC ⊆ Q. If Q ⊆ A, Q ⊆ B, C ⊆ Q and ABC ⊆ Q, Q is a prime quasi-PO-ternary ideal of T implies that A ⊆ Q or B ⊆ Q or C ⊆ Q. Therefore A = Q or B = Q or C = Q and hence Q is a weakly prime quasi-PO-ternary ideal of T.

Note IV.8: The converse of the theorem 4.7. need not be true. i.e., every weakly prime quasi-PO-ternary ideal of T is not prime quasi-PO-ternary ideal of T.

Example IV.9: Let T = $M_2(Z_0^-)$ is a PO-ternary semiring of 2 × 2 square matrices over Z_0^-. Let Q = $\left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in 30Z_0^- \right\}$ and $a \le b$ for $a, b \in Z_0^-$. Then Q is a weakly prime quasi-PO-ternary ideal of T. But Q is not a prime quasi-PO-ternary ideal of T. Since A = $\left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in 2Z_0^- \right\}$, B = $\left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in 3Z_0^- \right\}$ and C = $\left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in 5Z_0^- \right\}$ are quasi-PO-ternary ideal of T such that ABC ⊆ Q. But Q⊈ A, Q⊈ B and Q⊈C.

Theorem IV.10: Let T be a PO-ternary semiring and Q be a quasi-PO-ternary ideal of T. If Q is prime, then Q is left or lateral or right PO-ternary ideal of T.

Proof: Let Q be a prime quasi-PO-ternary ideal of T. Then (TTQ)(TQT ∪ TTQTT)(QTT) ⊆ TTQ ∩ (TQT ∪ TTQTT) ∩ QTT ⊆ Q. Since Q is prime, we have TTQ ⊆ Q or TQT ∩ TTQTT ⊆ Q or QTT ⊆ Q. therefore Q is left or lateral or right PO-ternary ideal of T.

Theorem IV.11: Let T be a commutative PO-ternary semiring and Q be a quasi-PO-ternary ideal of T. Then Q is prime if and only if *abc*∈ Q implies *a*∈ Q or *b*∈ Q or *c*∈ Q.

Proof :Suppose that Q is a prime quasi-PO-ternary ideal of T. Let *abc*∈ Q. Then by theorem4.10, Q is a PO-ternary ideal of T. Let $x \in \langle a\rangle_q \langle b\rangle_q \langle c\rangle_q$. Then *x* = ((*a*] ∩ (TT*a*]∩(T*a*T∪TT*a*TT] ∩ (*a*TT]).((*b*] ∩ (TT*b*] ∩(T*b*T∪TT*a*TT] ∩ (*b*TT]).((*c*] ∩ (TT*c*] ∩ (T*c*T∪TT*c*TT] ∩ (*c*TT]). Since *abc*∈ Q and Q is a PO-ternary ideal of T. Therefore *x*∈ Q. Thus $\langle a\rangle_q \langle b\rangle_q \langle c\rangle_q \subseteq$ Q. Since Q is prime quasi-PO-ternary ideal of T. Hence *a*∈ Q or *b*∈ Q or *c*∈ Q.
Converse is obvious.

Theorem IV.12: Let T be a PO-ternary semiring and Q be a quasi-PO-ternary ideal of T. Then Q is prime if and only if ((TT*a*] ∩ (T*a*T∪TT*a*TT] ∩ (*a*TT]).((TT*b*] ∩ (T*b*T∪TT*b*TT] ∩ (*b*TT]).((TT*c*] ∩ (T*c*T∪TT*c*TT] ∩ (*c*TT]) ⊆ Q implies *a*∈ Q or *b*∈ Q or *c*∈ Q.

Proof: Suppose that Q is a prime quasi-PO-ternary ideal of T and let ((TT*a*] ∩ (T*a*T∪TT*a*TT] ∩ (*a*TT]).((TT*b*] ∩ (T*b*T∪TT*b*TT] ∩ (*b*TT]).((TT*c*] ∩ (T*c*T∪TT*c*TT] ∩ (*c*TT]) ⊆ Q for some *a*, *b*, *c*∈ T. Clearly, ((TT*a*] ∩ (T*a*T∪TT*a*TT] ∩ (*a*TT]), ((TT*b*] ∩ (T*b*T∪TT*b*TT] ∩ (*b*TT]), ((TT*c*] ∩ (T*c*T∪TT*c*TT] ∩ (*c*TT]) are quasi-PO-ternary ideals of T.Since Q is prime, therefore ((TT*a*] ∩ (T*a*T∪TT*a*TT] ∩ (*a*TT])⊆ Q or ((TT*b*] ∩ (T*b*T∪TT*b*TT] ∩ (*b*TT]) ⊆ Q or ((TT*c*] ∩ (T*c*T∪TT*c*TT] ∩ (*c*TT])⊆ Q. If ((TT*a*] ∩ (T*a*T∪TT*a*TT] ∩ (*a*TT])⊆ Q, then $\langle a\rangle_q \subseteq$ Q implies that *a*∈ Q. Similarly, *b*∈ Q or *c*∈ Q.
Converse is obvious.

Theorem IV.13: Let T be a PO-ternary semiring. If the quasi-PO-ternary ideal of T with respect to inclusion relation form a chain, then every weakly prime quasi-PO-ternary ideal is a prime quasi-PO-ternary ideal of T.

Proof: Let Q be a weakly prime quasi-PO-ternary ideal of T. Let A, B, C are quasi-PO-ternary ideal of T such that ABC ⊆ Q. Suppose that A ⊈ Q, B ⊈ Q and C ⊈ Q. By the statement since Q ⊆ A, Q ⊆ B and Q ⊆ C. Since Q is weakly prime quasi-PO-ternary ideal of T. Therefore A = Q or B = Q or C = Q. This is a contradiction. Hence A ⊆ Q or B ⊆ Q or C ⊆ Q. Therefore Q is a prime quasi-PO-ternary ideal of T.

Theorem IV.14: Let T be a PO-ternary semiring. Then the following are equivalent

1. **The quasi-PO-ternary ideal of T is idempotent.**
2. **If A, B, C are three quasi-PO-ternary ideals of T such that A∩ B ∩ C ≠∅, then A ∩ B ∩ C = ABC.**
3. **$\langle a\rangle_q = [\langle a\rangle_q]^3$ for all *a*∈ T.**

Proof: (1) ⇒ (2): Suppose that A, B, C are quasi-PO-ternary ideals of T such that A∩B∩C ≠ ∅. Then by theorem III.11,A∩B∩C is a quasi-PO-ternary ideal of T. Since every quasi-PO-ternary ideal is an idempotent. Therefore (A∩B∩C) = $(A∩B∩C)^3$ = (A∩B∩C)(A∩B∩C)(A∩B∩C) ⊆ ABC.
(2) ⇒ (3): It is straight forward and (3) ⇒ (1) is obvious.

Definition IV.15: A non-empty subset A of a PO-ternary semiring T is said to be ***m_q-system*** provided for any *a*, *b*, *c*∈ A, there exist $x \in \langle a\rangle_q$, $y \in \langle b\rangle_q$, $z \in \langle c\rangle_q$ and *d* ∈ A such that $xyz \le d$.

Note IV.16: A non-empty subset A of a PO-ternary semiring T is called an *m_q-system* if *a*, *b*, *c*∈ A, there exist$x \in \langle a\rangle_q$, $y \in \langle b\rangle_q$, $z \in \langle c\rangle_q$ such that *xyz* ∈ (A].

Definition IV.17: A non-empty subset A of a PO-ternary semiring T is said to be ***n_q-system*** provided for any *a* ∈ A, there exist $x, y, z \in \langle a\rangle_q$and *d* ∈ A such that $xyz \le d$.

Note IV.18: A non-empty subset A of a PO-ternary semiring T is said to be ***n_q-system***provided for any *a*∈ A, there exist $x, y, z \in \langle a\rangle_q$ such that *xyz* ∈ (A].

Theorem IV.19: Every *m_q-system* is an*$_q$-system* of PO-ternary semiring T.

Proof: Suppose that the non-empty subset A of a PO-ternary semiring T is an *m_q-system*. Let for any *a*∈ A, there exist $x, y, z \in \langle a\rangle_q$. Since A is am*$_q$-system* and hence *xyz*∈(A]. therefore A is a *n_q-system* of T.

Note IV.20: The converse of the theorem 4.19, need not be true. i.e., every n_q-*system* of a PO-ternary semiring T need not be a m_q-*system* of T.

Example IV.21: Let T = Z_6^- is a PO-ternary semiring under usual addition, multiplication modulo 6 and natural ordering. Let A = {-2, -3}. Then A is an n_q-*system* but not an m_q-*system*.

Theorem IV.22: Let T be a PO-ternary semiring and Q is a quasi-PO-ternary ideal of T. Then Q is prime quasi-PO-ternary ideal of T if and only if T\Q is an m_q-*system* of T.

Proof: Suppose that Q is a prime quasi-PO-ternary ideal of T. Let $a, b, c \in$ T\Q.

Suppose that $xyz \nleq d$ for all $d \in$ T\Q and for all $x \in \langle a\rangle_q$, $y \in \langle b\rangle_q$ and $z \in \langle c\rangle_q$.

Then $\langle a\rangle_q\langle b\rangle_q\langle c\rangle_q \subseteq$ Q. Since Q is a prime quasi-PO-ternary ideal of T.

Therefore $a \in$ Q or $b \in$ Q or $c \in$ Q. This is a contradiction.

Therefore $xyz \leq d$ for some $x \in \langle a\rangle_q$, $y \in \langle b\rangle_q$ and $z \in \langle c\rangle_q$.

Hence T\Q is an m_q-*system* of T.

Conversely suppose that, A, B, C are quasi-PO-ternary ideals of T such that ABC ⊆ Q. Assume that A ⊈ Q, B ⊈ Q and C ⊈ Q. Let $a \in$ A\Q, $b \in$ B\Q and $c \in$ C\Q. Then $a, b, c \in$ T\Q. Since T\Q is an m_q-system. Therefore there exist an element $d \in$ T\Q such that $xyz \leq d$ for some $x \in \langle a\rangle_q$, $y \in \langle b\rangle_q$ and $z \in \langle c\rangle_q$. But $xyz \in \langle a\rangle_q\langle b\rangle_q\langle c\rangle_q \subseteq$ ABC ⊆ Q. This is a contradiction. Hence A ⊆ Q or B ⊆ Q or C ⊆ Q. Therefore Q is a prime quasi-PO-ternary ideal of T.

Theorem IV.23: Let T be a PO-ternary semiring and Q is a quasi-PO-ternary ideal of T. Then Q is semiprime quasi-PO-ternary ideal of T if and only if T\Q is an n_q-*system* of T.

Proof: Similar to the proof of the theorem IV.22.

Definition IV.24: A quasi-PO-ternary ideal of a PO-ternary semiring T is said to be ***T-prime quasi-PO-ternary ideal*** of T provided $xTyTz \subseteq$ Q implies $x \in$ Q or $y \in$ Q or $z \in$ Q.

Definition IV.25: A quasi-PO-ternary ideal of a PO-ternary semiring T is said to be ***T-semiprime quasi-PO-ternary ideal*** of T provided $xTxTx \subseteq$ Q implies $x \in$ Q.

Theorem IV.26: A quasi-PO-ternary ideal Q of a PO-ternary semiring T is T-prime if and only if RML ⊆Q implies R ⊆Q or M ⊆Q or L ⊆Q for any right PO-ternary ideal R, lateral PO-ternary ideal M and left PO-ternary ideal L of T.

Proof: Let Q be a T-prime quasi-PO-ternary ideal of T and RML ⊆ Q. Suppose R ⊈ Q and M ⊈ Q. Then there exist $x \in$ R\Q and $y \in$ M\Q. Let $z \in$ L. Then $xTyTz \subseteq$ RTMTL ⊆ RML ⊆ Q. Since Q is T-prime. Therefore, $x \in$ Q or $y \in$ Q or $z \in$ Q. But $x \notin$ Q and $y \notin$ Q. Hence $z \in$ Q and hence L ⊆ Q.

Conversely, suppose that $xTyTz \subseteq$ Q. Then $(xTT)(TyT)(TTz) \subseteq xTyTz \subseteq$ Q. Since xTT is a right PO-ternary ideal of T, TyT is a lateral PO-ternary ideal of T and TTz is a left PO-ternary ideal of T. Therefore, by hypothesis $xTT \subseteq$ Q or $TyT \subseteq$ Q or $TTz \subseteq$ Q. If $xTT \subseteq$ Q, then $x^3 \in xTT \subseteq Q$.

Now $\langle x\rangle_r\langle x\rangle_m\langle x\rangle_l = (x \cup xTT](x \cup TxT \cup TTxTT](x \cup TTx] \subseteq (x]^3 \cup (xTT] \subseteq$ Q. By hypothesis $\langle x\rangle_r \subseteq$ Q or $\langle x\rangle_m \subseteq$ Q or $\langle x\rangle_l \subseteq$ Q. Therefore $x \in$ Q. Similarly, if $TyT \subseteq$ Q ⇒ $y \in$ Q and if $TTz \subseteq$ Q ⇒ $z \in$ Q. Hence Q is T-prime PO-ternary ideal of T.

Notation IV.27: we use the following set defined as

L(Q) = {$x \in$ Q : (TTx] ⊆ Q},
M(Q) = { $x \in$ Q : (TxT∪TTxTT] ⊆ Q}
R(Q) = { $x \in$ Q : (xTT] ⊆ Q}
I_L = { $x \in$ L(Q) : (TTx] ⊆ L(Q)}
$_M I_M$ = { $x \in$ M(Q) : (TxT∪TTxTT] ⊆ M(Q)}
I_R = { $x \in$ R(Q) : (xTT] ⊆ R(Q)}.

Theorem IV.28: Let Q be a quasi-PO-ternary ideal of T. Then L(Q) is a left PO-ternary ideal of T contained in Q if Q is non-empty.

Proof: Let $x, y \in$ L(Q) and $s, t \in$ T. Then $x, y \in$ L(Q) ⇒ $stx \in (TTx] \subseteq$ Q, $sty \in (TTy] \subseteq$ Q.

⇒ $stx, sty \in$ Q ⇒ $stx + sty = st(x + y) \in (TT(x + y)] \subseteq Q$ ⇒ $x + y \in$ L(Q)

Now $TTstx \subseteq TTx$ ⇒ $(TTstx] \subseteq (TTx] \subseteq$ Q. Therefore $stx \in$ L(Q).

Consequently, TTL(Q) ⊆ Q. Hence L(Q) is a left PO-ternary ideal of T.

Theorem IV.29: Let Q be a quasi-PO-ternary ideal of T. Then M(Q) is a lateral PO-ternary ideal of T contained in Q if Q is non-empty.

Proof: Let $x, y \in$ M(Q) and $s, t \in$ T. Then $x, y \in$ M(Q) ⇒ $sxt \in (TxT] \subseteq$ Q, $syt \in (TyT] \subseteq$ Q.

⇒ $sxt, syt \in$ Q ⇒ $sxt + syt = s(x + y)t \in (T(x + y)T] \subseteq Q$
⇒ $x + y \in$ M(Q)

Now $TsxtT \subseteq TxT \cup TTxTT$ ⇒ $(TsxtT] \subseteq (TxT \cup TTxTT] \subseteq$ Q. Therefore $sxt \in$ M(Q).

Consequently, TM(Q)T∪ TTM(Q)TT ⊆ Q. Hence M(Q) is a lateral PO-ternary ideal of T.

Theorem IV.30: Let Q be a quasi-PO-ternary ideal of T. Then R(Q) is a right PO-ternary ideal of T contained in Q if Q is non-empty.

Proof: Let $x, y \in$ R(Q) and $s, t \in$ T. Then $x, y \in$ R(Q)
⇒ $xst \in (xTx] \subseteq$ Q, $yst \in (yTT] \subseteq$ Q.
⇒ $xst, yst \in$ Q ⇒ $xst + yst = (x + y)st \in ((x + y)TT] \subseteq Q$
⇒ $x + y \in$ R(Q)
Now $xstTT \subseteq xTT$ ⇒ $(xstTT] \subseteq (xTT] \subseteq$ Q.
Therefore $xst \in$ R(Q).
Consequently, R(Q)TT ⊆ Q.
Hence R(Q) is a right PO-ternary ideal of T.

Theorem IV.31: Let Q is a T-prime quasi-PO-ternary ideal of a PO-ternary semiring T. Then I_Q is a prime quasi-PO-ternary ideal of T.

Proof: Let Q be a T-prime quasi-PO-ternary ideal of a PO-ternary semiring T.

Suppose RML ⊆ I_Q for any PO-ternary ideals R, M and L of T.

Now $I_Q \subseteq$ L(Q) ⊆ Q implies RML ⊆ Q. Since Q is T-prime, therefore, by theorem 4.26, we have R ⊆ Q or M ⊆ Q or L ⊆ Q. Also I_Q is the largest PO-ternary ideal contained in Q, therefore, R ⊆ I_Q or M ⊆ I_Q or L ⊆ I_Q . Hence I_Q is a prime PO-ternary ideal of T.

Corollary IV.32: Let Q is a Semiprime quasi-PO-ternary ideal of a PO-ternary semiring T. Then I_Q is a Semiprime PO-ternary ideal of T.

Theorem IV.33: If a PO-ternary semiring T is a regular, then every quasi-PO-ternary ideal of T is T-semiprime.

Proof: Suppose that T is regular and Q be a quasi-PO-ternary ideal of T. Let $aTaTa\subseteq$ Q for $a\in$ T. Since T is regular, therefore, for $a\in$ T, there exist $x, y\in$ T such that $a\leq axaya$. Thus $a \leq axaya \Rightarrow a\in (aTaTa] \subseteq aTaTa \subseteq$ Q.Therefore $a\in$ Q. Hence Q is a T-semiprime.

V. QUASI-*K*-PO-TERNARY IDEALS

Definition V.1: An additive subsemigroup Q of a PO-ternary semiring T is said to be ***quasi-k-PO-ternary ideal*** of T provided $\overline{QTT}\cap(\overline{TQT}\cup\overline{TTQTT})\cap\overline{TTQ}\subseteq Q$ and (Q] ⊆ Q.

Theorem V.2: Let T be a PO-ternary semiring and A, B, C ⊆ T. Then $\overline{ABC}=\overline{\overline{A}\,\overline{B}\,\overline{C}}$.

Proof: Since A $\subseteq \overline{A}$, B $\subseteq \overline{B}$ and C $\subseteq \overline{C}$, therefore, ABC $\subseteq \overline{A}\,\overline{B}\,\overline{C}$. Hence $\overline{ABC}\subseteq\overline{\overline{A}\,\overline{B}\,\overline{C}}$. Again, let $x\in \overline{A}$, $y\in \overline{B}$ and $z\in \overline{C}$. Then there exist $a_1, a_2\in$ A, $b_1, b_2\in$ B and $c_1, c_2\in$ C such that $x + a_1 = a_2$, $y + b_1 = b_2$ and $z + c_1 = c_2$. Now

$xyz+ a_2b_2c_1 + a_2b_1c_2 + a_1b_2c_2 + a_1b_1c_1$

$= xyz + (x + a_1)(y + b_1)c_1 + a_2b_1c_2 + a_1b_2c_2 + a_1b_1c_1$

$= xyz + xyc_1 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + a_2b_1c_2+a_1b_2c_2 + a_1b_1c_1$

$= xyc_2 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + a_2b_1c_2+a_1b_2c_2 + a_1b_1c_1$

$= xyc_2 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + (x + a_1)b_1c_2+a_1b_2c_2 + a_1b_1c_1$

$= x(y + b_1)c_2 + xb_1c_1 + a_1(y + b_1)c_1 + a_1b_1c_2+a_1b_2c_2 + a_1b_1c_1$

$= xb_2c_2 + (x + a_1)b_1c_1 + a_1b_2c_1 + a_1b_1c_2 + a_1b_2c_2$

$= (x + a_1)b_2c_2 + a_2b_1c_1 + a_1b_2c_1 + a_1b_1c_2= a_2b_2c_2 + a_2b_1c_1 + a_1b_2c_1 + a_1b_1c_2$.

As $a_ib_ic_i\in$ ABC, where i = 1, 2. Therefore we can prove that $xyz\in \overline{ABC}$ for $x\in \overline{A}$, $y\in \overline{B}$ and $z\in \overline{C}$. Suppose that $t\in \overline{A}\,\overline{B}\,\overline{C}$. Then $t = \sum_{i=}^{n} a_ib_ic_i$ for some $a_i\in \overline{A}$, $b_i\in \overline{B}$, $c_i\in \overline{C}$. Thus t= $\overline{ABC}$. Therefore $\overline{A}\,\overline{B}\,\overline{C}\subseteq\overline{ABC}$. Hence $\overline{\overline{A}\,\overline{B}\,\overline{C}}\subseteq\overline{\overline{ABC}}=\overline{ABC}$. Therefore $\overline{\overline{A}\,\overline{B}\,\overline{C}}=\overline{ABC}$.

Definition IV.3: A PO-ternary semiring T is said to be ***k-regular*** provided for each a ∈ T there exist $x, y\in$ T such that $a + axa = aya$.

Theorem IV.4: If a PO-ternary semiring T is *k*-regular. Then every quasi-*k*-PO-ternary ideal Q of T is of the form Q = $\overline{QTQTQ}=\overline{TTQ}\cap(\overline{TQT}\cup\overline{TTQTT})\cap\overline{QTT}$.

Proof: Let Q be a quasi-*k*-PO-ternary ideal of T. Then $\overline{QTT}\cap(\overline{TQT}\cup\overline{TTQTT})\cap\overline{TTQ}\subseteq Q$ and (Q] ⊆ Q. Let $a\in$ Q and T is *k*-regular, then there exist $x, y\in$ T such that $a + axa = aya \Rightarrow axa + axaxa = ayaxa$. Since $axaxa$, $ayaxa\in$ QTQTQ. Therefore $axa\in \overline{QTQTQ}$. Similarly, $aya\in \overline{QTQTQ}$. Since $\overline{QTQTQ}$ is *k*-closed and hence $a\in \overline{\overline{QTQTQ}}=\overline{QTQTQ}$. Therefore Q $\subseteq \overline{QTQTQ}$. Again QTQTQ ⊆Q(TTT)T ⊆ QTT and QTQTQ ⊆ TTQ and QTQTQ ⊆ TTQTT and hence $\overline{QTQTQ}\subseteq\overline{TTQ}$, $\overline{QTQTQ}\subseteq\overline{QTT}$ and $\overline{QTQTQ}\subseteq\overline{TQT}\cup\overline{TTQTT}$ as $0\subseteq\overline{TQT}$.

Thus we have Q $\subseteq\overline{QTQTQ}\subseteq\overline{QTT}\cap(\overline{TQT}\cup\overline{TTQTT})\cap\overline{TTQ}\subseteq Q$ as Q is quasi-*k*-PO-ternary ideal of T. Hence Q = $\overline{QTQTQ}$ = $\overline{QTT}\cap(\overline{TQT}\cup\overline{TTQTT})\cap\overline{TTQ}$.

VI. CONCLUSION

In this paper mainly we studied about quasi po-*k*-ternary ideals and full quasi po-*k*-ternary ideals in PO-ternary semiring.

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