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Exponential Metric in Binary Mass Systems Gravitational Field

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***Abstract*— This paper deals with the procedure for determining binary mass systems metric. It is performed by placing a test mass m in the common gravitational field of two other objects having much larger mass. The resulting gijmetric tensor is diagonal with the exponential component. Metric tensor exponential components show that the metric in binary systems gravitational field equals the product of metrics of objects making up the system.**

***Index Terms*— exponential metrics, mass systems.**

I. Introduction

A relativistic approach in the study of the gravitational field of double stars, or binary systems of heavy masses, leads to exponential metric as an apparent metric of Newtonian gravity.

This theoretical concept is based on the application of Newton's law of gravity using the Einstein mass-energy equivalence, i.e. $E=mc^2$ [1,2,3].

This is achievedby considering the relativistic motion of a body of point mass **m** in the gravitational field of the binary system of masses M_1 and M_2, where m << M_1i m << M_2.

The position of an arbitrary point **P** in which the body of mass **m** location is determined by the radius vector in the Cartesian coordinate system, which is moved to the center of mass M_1 and M_2 (Figure 1),

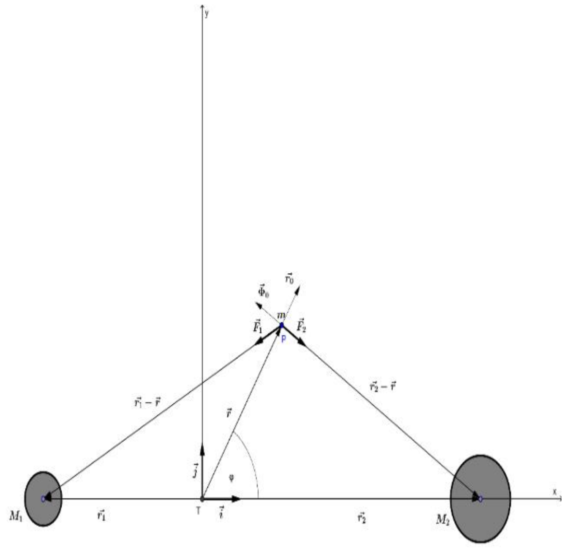

Figure 1

The gravitational field of mass M_1 at point Pis

$$\overrightarrow{g_1} = \frac{R_1 c^2}{|\vec{r}_1 - \vec{r}|^3}(\vec{r}_1 - \vec{r}) \quad (1)$$

while the gravitational field of mass M_2:

$$\overrightarrow{g_2} = \frac{R_2 c^2}{|\vec{r}_2 - \vec{r}|^3}(\vec{r}_2 - \vec{r}) \quad (2)$$

The gravitational radii R_1 and R_2 are defined by the expressions:

$$R_1 = \frac{GM_1}{c^2} \quad (3a)$$

$$R_2 = \frac{GM_2}{c^2} \text{(3b)}$$

The total gravitational field at point **P** will be the vector sum of the gravitational fields,$\overrightarrow{g_1} i \overrightarrow{g_2}$ i.e.

$$\overrightarrow{g_R} = \overrightarrow{g_1} + \overrightarrow{g_2} \quad (4)$$

The resultant gravitational force acting on mass **m** at point **P** is

$$\vec{F}_R = m\overrightarrow{g_R} = mc^2\left[\frac{R_1(\vec{r}_1 - \vec{r})}{|\vec{r}_1 - \vec{r}|^3} + \frac{R_2(\vec{r}_2 - \vec{r})}{|\vec{r}_2 - \vec{r}|^3}\right] \text{(5)}$$

II. Binary mass system energy determination

The potential energy change of M_1 and M_2coupled system equalsthe work of force$\vec{F}_R$ in the gravitational field $\vec{g}_R$,i.e.

$$dU = -\vec{F}_R \cdot d\vec{r} = -\vec{F}_R\left(\vec{r}_0 d\vec{r} + \mathrm{r}\vec{\phi}_0 d\varphi\right)(6)$$

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Using relation (5),equation (6) is obtained:

$$dU = -mc^2\left[R_1\frac{-r_1\cos(\varphi)dr + r_1 r\sin(\varphi)d\varphi - rdr}{r_1{}^3\left(1-\frac{2r}{r_1}\cos(\varphi)+\frac{r^2}{r_1{}^2}\right)^{3/2}} + R_2\frac{r_2\cos(\varphi)dr - r_2 r\sin(\varphi)d\varphi - rdr}{r_2{}^3\left(1+\frac{2r}{r_2}\cos(\varphi)+\frac{r^2}{r_2{}^2}\right)^{3/2}}\right](7)$$

Since

$$c^2 dm = -dU = -\frac{\partial U}{\partial r}dr - \frac{\partial U}{\partial \varphi}d\varphi(8)$$

it follows that

$$d(\ln m) = P(r,\varphi)dr + Q(r,\varphi)d\varphi(9)$$

is

$$P(r,\varphi) = -\frac{R_1 r_1{}^2\left(\frac{\cos(\varphi)}{r_1}+\frac{r}{r_1{}^2}\right)}{r_1{}^3\left(1+\frac{2r}{r_1}\cos(\varphi)+\frac{r^2}{r_1{}^2}\right)^{3/2}} - \frac{R_2 r_2{}^2\left(-\frac{\cos(\varphi)}{r_2}+\frac{r}{r_2^2}\right)}{r_1{}^3\left(1-\frac{2r}{r_2}\cos(\varphi)+\frac{r^2}{r_2{}^2}\right)^{3/2}}(10a)$$

$$Q(r,\varphi) = \frac{R_1 r_1 r\sin(\varphi)}{r_1{}^3\left(1-\frac{2r}{r_1}\cos(\varphi)+\frac{r^2}{r_1{}^2}\right)^{3/2}} - \frac{R_2 r_2 r\sin(\varphi)}{r_1{}^3\left(1+\frac{2r}{r_2}\cos(\varphi)+\frac{r^2}{r_2{}^2}\right)^{3/2}}(10b)$$

Integrationof equation (9) gives

$$m(r,\varphi) = m_\infty e^{\frac{R_1}{r_1\sqrt{1+\frac{2r}{r_1}\cos(\varphi)+\frac{r^2}{r_1{}^2}}}+\frac{R_2}{r_2\sqrt{1-\frac{2r}{r_2}\cos(\varphi)+\frac{r^2}{r_2{}^2}}}}(11)$$

Where $m(r,\varphi) = m_\infty$, when $\vec{r} \to \infty$ [4,5].
By introducing the following function

$$f(r,\varphi) = \frac{R_1}{r_1\sqrt{1+\frac{r^2}{r_1{}^2}-\frac{2r}{r_1}\cos(\varphi)}} + \frac{R_2}{r_2\sqrt{1+\frac{r^2}{r_2{}^2}+\frac{2r}{r_2}\cos(\varphi)}}(12)$$

equation (11) is written in the form

$$m = m_\infty e^{f(r,\varphi)}(13)$$

because

$$\frac{\partial f(r,\varphi)}{\partial r} = P(r,\varphi)\ (14a)$$

$$\frac{\partial f(r,\varphi)}{\partial \varphi} = Q(r,\varphi)\ (14b)$$

The total differential of the function is

$$df(r,\varphi) = P(r,\varphi)dr + Q(r,\varphi)d\varphi(15)$$

The change in gravitational potential of energy systems is defined using equations (8) and (13), giving

$$dU = -d\left[m_0 c^2 e^{f(r,\varphi)}\right](16)$$

By integrating the equations using the realconditions$U(\infty) = 0$, when $\vec{r} \to \infty$, the gravitational potential energy of the binary mass system is obtained:

$$U(r,\varphi) = -mc^2\left[1 - e^{-f(r,\varphi)}\right](17)$$

The total energy differential of the system is equal to the sum of the differentials of kinetic and gravitational potential energy:

$$dE = dE_k + dU = -d[mc^2 - mc^2(1 - e^{-f(r,\varphi)})] = d[mc^2 e^{-f(r,\varphi)})](18)$$

By grouping equations (18), the total energy of the binary system is obtained:

$$E = mc^2 e^{-f(r,\varphi)}(19)$$

Applying the relativistic relations between energy and momentum, the total energy (19) through the Hamiltonian is as follows

$$H(\vec{r},\vec{p}) = e^{-f(r,\varphi)}\sqrt{g_{\alpha\beta}p^{\alpha}p^{\beta}c^2 + \mathrm{m}_0^2 c^4}(20)$$
$$\alpha,\beta = 1,2,3$$

From the Hamiltonian canon equation

$$\frac{\partial H(\vec{r},\vec{p})}{\partial p^{\alpha}} = \dot{q}_{\alpha} = \frac{p_{\alpha}}{m}e^{-f(r,\varphi)}(21)$$

the momentum p_{α}, is obtained, i.e.

$$p_{\alpha} = m\dot{q}_{\alpha}e^{f(r,\varphi)}(22)$$

It is possible to examine the description of a binary system using a Lagrange formalism whose base consists of generalized coordinates:

$$L(q^i,\dot{q}^i) = -m_0 c\frac{ds}{dt} = -m_0 c\sqrt{-g_{ij}\frac{dq^i}{dt}\frac{dq^j}{dt}}(23)$$
$$i,j = 0,1,2,3$$

and by applying the Lagrange transform:

$$p_{\alpha}\dot{q}_{\alpha} - H(q^{\alpha},p^{\alpha}) = L(q^{\alpha},\dot{q}_{\alpha})(24)$$
$$\alpha = 1,2,3$$

a line element is obtained

$$ds^2 = c^2dt^2e^{-2f(r,\varphi)} - q_{\alpha\beta}dq^{\alpha}dq^{\beta}(25)$$

The metric tensor $\mathbf{g_{ij}}$is the diagonal whose components are

$$g_{0\alpha} = 0$$
$$g_{00} = -e^{-2f(r,\varphi)}(26)$$
$$g_{11} = e^{2f(r,\varphi)}$$
$$g_{22} = r^2e^{2f(r,\varphi)}$$

The Yilmaz metric has a linear element

$$ds^2 = e^{\Phi}c^2dt^2 - e^{-\Phi}(dx^2 + dy^2 + dz^2)(27)$$

If the function f (r, φ) according to equation (12) is written in the form:

$$f(r,\varphi) = f_1(r,\varphi) + f_2(r,\varphi)(28)$$

then

$$f_1(r,\varphi)\frac{R_1}{r_1\sqrt{1-\frac{2r}{r_1}\cos(\varphi)+\frac{r^2}{{r_1}^2}}}(29a)$$

$$f_2(r,\varphi) = \frac{R_2}{r_2\sqrt{1+\frac{2r}{r_2}\cos(\varphi)+\frac{r^2}{{r_2}^2}}}(29b)$$

then the components of theg_{ij}metric tensor can be written in the form:

$$g_{0\alpha} = 0$$
$$g_{00} = -e^{-2f_1(r,\varphi)}e^{-2f_2(r,\varphi)}$$
$$g_{11} = e^{2f_1(r,\varphi)}e^{2f_2(r,\varphi)}(30)$$
$$g_{22} = r^2e^{2f_1(r,\varphi)}e^{2f_2(r,\varphi)}$$

From equations (28), it can be concluded that the metric of linear mass systems is equal to the product of metrics of the system components.

III. Conclusion

A relativistic approach in the study of the gravitational field of binary meat systems leads to an exponential metric. Metric results obtained, $g_{ij}$is diagonal.Diagonal metric tensor components show that the metric in the gravitational field of binary systems is equal to the product of the metric of the system components.

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