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A Study on Vibration Reduction of the Torsional-Vibration Responses of A Uniform Beam Subjected To An Eccentric Moving Force

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Abstract—**The conventional single degree-of-freedom (dof) vibration absorber is a widely used device for suppressing the (vertical or horizontal) flexural-vibration responses of a structure induced by dynamic loads. Thus, the relating information is abundant. However, because the device for suppressing the torsional-vibration responses of a structure is rare, the relating literature is limited. The objective of this paper is to determine the optimal parameters for a two-dof vibration absorber such that the torsional-vibration responses of a structure due to dynamic loads can be effectively suppressed. To this end, the equations of motion for a two-dof vibration absorber and the generalized main vibration system corresponding to a certain predominant mode shape of the beam will be firstly derived using the theory of Lagrange's equations. Then, based on the dynamic magnification factor for the rotational dof of the last generalized main vibration system, the optimal parameters of the aforementioned two-dof vibration absorber will be determined. Numerical investigations concerning the use of two-dof vibration absorber for suppressing the rotational-vibration responses of the generalized main vibration system due to dynamic loads will then be conducted. Finally, based on the mode superposition principle, the foregoing technique will be extended to suppress the torsional-vibration responses of a uniform beam subjected to an eccentric moving force. Because, in addition to the torsional-vibration responses, the presented two-dof vibration absorber can also suppress the flexural-vibration responses of the structure to some degree, its functions should be greater than those of the conventional single-dof absorber.**

Index Terms—**Dynamic magnification factor, Dynamic loads, Single degree-of-freedom vibration absorber, Torsional vibration, Two degree-of-freedom vibration absorber.**

I. Introduction

To suppress the structural vibration responses induced by dynamic loads, the dynamic characteristics of vibration absorbers have been investigated by many researchers [1-12]. For example, Brock and Mo [1], Warburton [2,3], Hartog [4], Ormondroyd and Hartog [5] have investigated the optimization of vibration absorber for suppressing the (vertical and horizontal) flexural-vibration responses of the single degree-of-freedom (dof) spring-mass main system, where the absorber was the type of one-dof spring-damper-mass system and directly attached to the main system. It is well known that any real structure can be reasonably represented as a multiple dof system. However, if the contribution of a particular mode to its dynamic responses is much more significant than those of the other modes, then the last multiple dof system can be simplified as an one-dof spring-mass main system. Therefore, in spite of that the optimal absorber parameters presented in references [1-5] are for the case of one-dof spring-mass main system, the last absorber parameters can also be used for the case of multiple dof structure according to its simplified one-dof spring-mass main system. Based on the last concept, Joshi and Jangid [6], Kwon *et al.* [7], Yau and Yang [8], Rana and Soong [9] and Rice [10] have used the one-dof vibration absorber for suppressing the flexural-vibration responses of the multiple dof structures subjected to earthquake loadings and moving loads. Besides, Sun *et al.* [11] and Xue *et al.* [12] have used a tuned liquid column damper for suppressing the torsional motions of structures. From the review of the above-mentioned literature, one finds that the reports [1-10] concerning the use of one-dof vibration absorber for suppressing the (vertical or horizontal) flexural-vibration responses of structures are plenty, however, those for suppressing the torsional ones are limited [11-12]. Because the information concerning the use of *two-dof vibration absorber* (rather than tuned liquid column damper presented in [11] and [12]) for suppressing torsional-vibration responses of structures (cf. Figure 1) is not found so far, it is studied in this paper.

Firstly, by means of the Lagrange's equations, the equations of motion of the generalized main vibration system corresponding to a certain predominant mode shape of the beam and the attached two-dof vibration absorber (cf. Figure 2) are derived. Then, the optimal parameters of a two-dof vibration absorber for suppressing the rotational-vibration responses of the generalized main vibration system due to an external harmonic torque are determined. Next, based on the modal data obtained from the mode superposition methodology and the orthogonal property between the normal mode shapes of the multiple dof beam, the technique for determining the optimal absorber parameters associated with any order of vibration mode of the beam is presented. Finally, the last optimal absorber parameters are used for suppressing the torsional-vibration responses of a pinned-pinned beam subjected to an eccentric moving force (cf. Figure 5). Numerical results show that the presented optimal parameters for the two-dof vibration absorber do provide a technique for effectively suppressing the torsional-vibration responses of structures. Due to the fact that the presented absorber not only suppresses the torsional-vibration responses of structures but

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also the (vertical and harizontal) flexural-vibration ones, the presented two-dof vibration absorber should be better than the conventional one-dof absorber.

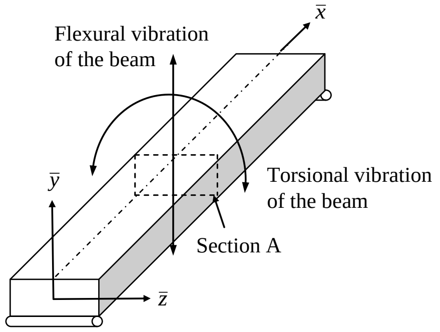

Figure 1 Flexural vibration and torsional vibration of a beam.

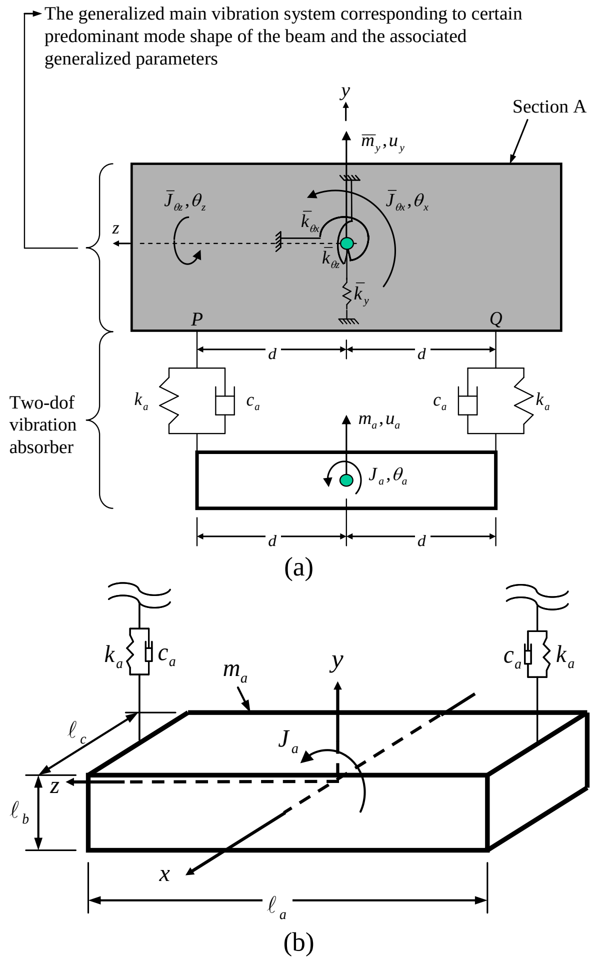

Figure 2 (a) The generalized main vibration system corresponding to certain predominant mode shape of the beam and the associated generalized parameters, and (b) mathematical model of the two-dof vibration absorber.

II. EQUATIONS OF MOTION OF THE GENERALIZED MAIN VIBRATION SYSTEM AND THE TWO-DOF VIBRATION ABSORBER

In order to suppress the torsional-vibration responses of the beam shown in Figure 1, a two-dof vibration absorber is installed to Section A of the beam. Figure 2(a) shows the generalized main vibration system corresponding to certain predominant mode shape of the beam and the associated generalized parameters, and the attached two-dof vibration absorber. In the figure, $\bar{m}_y$, $\bar{J}_{\theta x}$ and $\bar{J}_{\theta z}$ are the mass and masses moment of inertia about the x and z axes of the generalized main vibration system, respectively; $\bar{k}_y$, $\bar{k}_{\theta x}$ and $\bar{k}_{\theta z}$ are the translational (y) spring constant and rotational spring constants about the x and z axes of the generalized main vibration system, respectively; u_y, θ_x and θ_z are the instantaneous translational (y) displacement and rotational angles about the x and z axes of the generalized main vibration system, respectively; u_a and θ_a are instantaneous translational (y) displacement and rotational angle about the x axis of the absorber, while d is the distance between the two springs and the lumped mass m_a (or $\bar{m}_y$). In addition, P and Q are the contact points between the generalized main vibration system and the absorber, while k_a, c_a, m_a and J_a are the spring constant, damping coefficient, mass and mass moment of inertia of the absorber, respectively. It is evident that the motions of a two-dof vibration absorber are limited to the yz plane and the rotational motions of the generalized main vibration system about z axis have nothing to do with the two-dof vibration absorber. For this reason, the rotational motions of the generalized main vibration system about z axis are neglected in the following formulas.

The translational (y) displacements of the contact points, P and Q, are given by

$$u_P = u_y - d\theta_x \tag{1}$$

$$u_Q = u_y + d\theta_x \tag{2}$$

The total kinetic energy (T) and potential energy (V) of the generalized main vibration system together with the two-dof vibration absorber are:

$$T = \frac{1}{2}\bar{m}_y\dot{u}_y^{\,2} + \frac{1}{2}\bar{J}_{\theta x}\dot{\theta}_x^{\,2} + \frac{1}{2}m_a\dot{u}_a^{\,2} + \frac{1}{2}J_a\dot{\theta}_a^{\,2} \tag{3}$$

$$V = \frac{1}{2}\bar{k}_{\theta x}\theta_x^2 + \frac{1}{2}\bar{k}_y u_y^2 + \frac{1}{2}k_a[u_P - (u_a - d\theta_a)]^2 + \frac{1}{2}k_a[u_Q - (u_a + d\theta_a)]^2 \tag{4}$$

The generalized forces of the generalized main vibration system and the two-dof vibration absorber in the translational (y) and rotational (θ_x) directions are respectively given by

$$F_{u_y} = -2c_a(\dot{u}_y - \dot{u}_a) + f_y \tag{5}$$

$$F_{\theta_x} = -2c_a d^2(\dot{\theta}_x - \dot{\theta}_a) + f_{\theta x} \tag{6}$$

$$F_{u_a} = 2c_a(\dot{u}_y - \dot{u}_a) + f_a \tag{7}$$

$$F_{\theta_a} = 2c_a d^2(\dot{\theta}_x - \dot{\theta}_a) + f_{\theta a} \tag{8}$$

where f_y and $f_{\theta x}$ are the external force and torque applied on the generalized main vibration system, while f_a and $f_{\theta a}$ are those applied on the absorber.

Substituting Equations (1)-(8) into the following Lagrange's equations [13]

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\lambda}}\right) - \frac{\partial T}{\partial \lambda} + \frac{\partial V}{\partial \lambda} = F_\lambda \quad (\lambda = u_y, \theta_x, u_a, \theta_a) \tag{9}$$

one obtains

$$\bar{m}_y\ddot{u}_y + 2k_a u_y - 2k_a u_a + \bar{k}_y u_y = -2c_a\dot{u}_y + 2c_a\dot{u}_a + f_y \quad (10)$$

$$\bar{J}_{\theta x}\ddot{\theta}_x + 2k_a d^2\theta_x - 2k_a d^2\theta_a + \bar{k}_{\theta x}\theta_x = -2c_a d^2\dot{\theta}_x + 2c_a d^2\dot{\theta}_a + f_{\theta x} \quad (11)$$

$$m_a\ddot{u}_a - 2k_a u_y + 2k_a u_a = 2c_a\dot{u}_y - 2c_a\dot{u}_a + f_a \quad (12)$$

$$J_a\ddot{\theta}_a - 2k_a d^2\theta_x + 2k_a d^2\theta_a = 2c_a d^2\dot{\theta}_x - 2c_a d^2\dot{\theta}_a + f_{\theta a} \quad (13)$$

Writing Equations (10)-(13) in matrix form yields

$$[m]\{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{f\} \quad (14)$$

where

$$\{q\} = [u_y \quad \theta_x \quad u_a \quad \theta_a]^T \quad (15)$$

$$\{\dot{q}\} = [\dot{u}_y \quad \dot{\theta}_x \quad \dot{u}_a \quad \dot{\theta}_a]^T \quad (16)$$

$$\{\ddot{q}\} = [\ddot{u}_y \quad \ddot{\theta}_x \quad \ddot{u}_a \quad \ddot{\theta}_a]^T \quad (17)$$

$$\{f\} = [f_y \quad f_{\theta x} \quad f_a \quad f_{\theta a}]^T \quad (18)$$

$$[m] = \begin{bmatrix} \bar{m}_y & 0 & 0 & 0 \\ 0 & \bar{J}_{\theta x} & 0 & 0 \\ 0 & 0 & m_a & 0 \\ 0 & 0 & 0 & J_a \end{bmatrix} \quad (19)$$

$$[c] = \begin{bmatrix} 2c_a & 0 & -2c_a & 0 \\ 0 & 2c_a d^2 & 0 & -2c_a d^2 \\ -2c_a & 0 & 2c_a & 0 \\ 0 & -2c_a d^2 & 0 & 2c_a d^2 \end{bmatrix} \quad (20)$$

$$[k] = \begin{bmatrix} 2k_a + \bar{k}_y & 0 & -2k_a & 0 \\ 0 & 2k_a d^2 + \bar{k}_{\theta x} & 0 & -2k_a d^2 \\ -2k_a & 0 & 2k_a & 0 \\ 0 & -2k_a d^2 & 0 & 2k_a d^2 \end{bmatrix} \quad (21)$$

Equation (14) is the equation of motion of the generalized main vibration system and the two-dof vibration absorber. In addition, Equations (10) and (12) represent the equations of motion of the generalized main vibration system and the two-dof vibration absorber in the translational (y) direction, while Equations (11) and (13) represent those in the rotational (θ_x) direction.

In order to calculate the dynamic responses of a beam carrying a two-dof vibration absorber subjected to an eccentric moving force by conventional finite element method, the property matrices of a two-dof vibration absorber are also presented in the following.

By setting $\bar{m}_y = \bar{J}_{\theta x} = \bar{k}_y = \bar{k}_{\theta x} = 0$ in Equations (19)-(21), one obtains

$$[m_a] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_a & 0 \\ 0 & 0 & 0 & J_a \end{bmatrix} \quad (22)$$

$$[c_a] = \begin{bmatrix} 2c_a & 0 & -2c_a & 0 \\ 0 & 2c_a d^2 & 0 & -2c_a d^2 \\ -2c_a & 0 & 2c_a & 0 \\ 0 & -2c_a d^2 & 0 & 2c_a d^2 \end{bmatrix} \quad (23)$$

$$[k_a] = \begin{bmatrix} 2k_a & 0 & -2k_a & 0 \\ 0 & 2k_a d^2 & 0 & -2k_a d^2 \\ -2k_a & 0 & 2k_a & 0 \\ 0 & -2k_a d^2 & 0 & 2k_a d^2 \end{bmatrix} \quad (24)$$

where $[m_a]$, $[c_a]$ and $[k_a]$ given by Equations (22)-(24) are, respectively, the mass, damping and stiffness matrices of the two-dof vibration absorber.

It is noted that the mass moment of inertia of the two-dof vibration absorber (J_a) has close relationship with the magnitude and size of the lumped mass of the absorber. If the magnitude and size for the lumped mass of the absorber are m_a and $\ell_a \times \ell_b \times \ell_c$, respectively (see Figure 2(b)), then its mass moment of inertia about the x axis is given by [14]

$$J_a = \frac{1}{12} m_a (\ell_a^2 + \ell_b^2) \quad (25)$$

where ℓ_a, ℓ_b and ℓ_c are the length, height and width of the lumped mass m_a, respectively.

III. DYNAMIC MAGNIFICATION FACTOR OF THE GENERALIZED MAIN VIBRATION SYSTEM

Because the aim of this section is to find the optimal absorber parameters for suppressing the rotational motions of the generalized main vibration system, only Equations (11) and (13) will be discussed hereafter. It is worthy of mention that, if the aim is to suppress the translational (y) responses of the generalized main vibration system, one must design the absorber according to Equations (10) and (12).

If the generalized main vibration system is subjected to a harmonic torque in the rotational (θ_x) direction (cf. Figure 2(a)) and the external torque applied on the two-dof vibration absorber is zero, one has

$$f_{\theta x} = \bar{f}_{\theta x} e^{j\omega t} \quad (26)$$

$$f_{\theta a} = 0 \quad (27)$$

where $\bar{f}_{\theta x}$ and ω represent the amplitude and forcing frequency of the harmonic torque $f_{\theta x}$, respectively.

In such a case, the steady-state rotational angles of the generalized main vibration system and the absorber, θ_x and θ_a, take the form

$$\theta_x = \bar{\theta}_x e^{j\omega t} \quad (28)$$

$$\theta_a = \bar{\theta}_a e^{j\omega t} \quad (29)$$

where $\bar{\theta}_x$ and $\bar{\theta}_a$ are the amplitudes of θ_x and θ_a, respectively, and $j = \sqrt{-1}$.

Time derivatives of Equations (28) and (29) give

$$\dot{\theta}_x = j\omega\theta_x, \quad \ddot{\theta}_x = -\omega^2\theta_x \quad (30)$$

$$\dot{\theta}_a = j\omega\theta_a, \quad \ddot{\theta}_a = -\omega^2\theta_a \quad (31)$$

Substituting Equations (26)-(31) into Equations (11) and (13) gives

$$(-\bar{J}_{\theta x}\omega^2+\bar{k}_{\theta x}+2k_a d^2+2j\omega c_a d^2)\bar{\theta}_x -(2j\omega c_a d^2+2k_a d^2)\bar{\theta}_a=\bar{f}_{\theta x} \quad (32)$$

$$-(2k_a d^2+2j\omega c_a d^2)\bar{\theta}_x +(2j\omega c_a d^2+2k_a d^2-J_a\omega^2)\bar{\theta}_a=0 \quad (33)$$

Because the aim of this paper is to suppress the rotational-vibration responses (θ_x) of the generalized main vibration system to the possible extent, only the response amplitude $\bar{\theta}_x$ is interested here. Solving the last two equations for $\bar{\theta}_x$, one has

$$\bar{\theta}_x=\frac{A_1+jA_2}{(A_1A_3-A_4)+j[A_2(A_3-J_a\omega^2)]}\bar{f}_{\theta x} \quad (34)$$

where

$$A_1=2k_a d^2-J_a\omega^2 \quad (35a)$$

$$A_2=2\omega c_a d^2 \quad (35b)$$

$$A_3=\bar{k}_{\theta x}-\bar{J}_{\theta x}\omega^2 \quad (35c)$$

$$A_4=2k_a d^2 J_a\omega^2 \quad (35d)$$

Because $\bar{\theta}_x$ is a complex number, its magnitude is

$$\left|\bar{\theta}_x\right|=\sqrt{\frac{A_1^2+A_2^2}{(A_1A_3-A_4)^2+[A_2(A_3-J_a\omega^2)]^2}}\bar{f}_{\theta x} \quad (36)$$

Therefore, the dynamic magnification factor for rotational-vibration responses (θ_x) of the generalized main vibration system is

$$\left|\frac{\bar{\theta}_x}{\theta_{st}}\right|=\sqrt{\frac{A_1^2+A_2^2}{(\frac{A_1A_3-A_4}{\bar{k}_{\theta x}})^2+[A_2(\frac{A_3-J_a\omega^2}{\bar{k}_{\theta x}})]^2}} \quad (37)$$

where

$$\theta_{st}=\frac{\bar{f}_{\theta x}}{\bar{k}_{\theta x}} \quad (38)$$

IV. Non-dimensional Optimal Frequency Ratio and Damping Ratio of the Two-dof Vibration Absorber

Multiplying the numerator and denominator inside the square root of Equation (37) with $(\bar{J}_{\theta x}/J_a\bar{k}_{\theta x})^2$, respectively, yields

$$\left|\frac{\bar{\theta}_x}{\theta_{st}}\right|=\sqrt{\frac{R}{S}} \quad (39)$$

where

$$R=(f^2-\Omega^2)^2+(2\xi f\,\Omega)^2 \quad (40)$$

$$S=[(1-\Omega^2)(f^2-\Omega^2)-\mu f^2\Omega^2]^2 +(2\xi f\,\Omega)^2(1-\mu\,\Omega^2-\Omega^2)^2 \quad (41)$$

In the last equations, the damping ratio (ξ), frequency ratios (f and Ω) and the ratio for mass moment of inertia (μ) are defined in the following.

$$\xi=\frac{2c_a d^2}{2\sqrt{2J_a k_a d^2}} \quad (42)$$

$$f=\frac{\sqrt{2k_a d^2/J_a}}{\sqrt{\bar{k}_{\theta x}/\bar{J}_{\theta x}}} \quad (43)$$

$$\Omega=\frac{\omega}{\sqrt{\bar{k}_{\theta x}/\bar{J}_{\theta x}}} \quad (44)$$

$$\mu=\frac{J_a}{\bar{J}_{\theta x}} \quad (45)$$

From Equation (39), one may obtain the optimal frequency and damping ratios of the two-dof vibration absorber [4]:

$$f_{opt}=\frac{1}{1+\mu} \quad (46)$$

$$\xi_{opt}=\sqrt{\frac{3\mu}{8(1+\mu)}} \quad (47)$$

For the detailed procedures of determining the last two optimal parameters of the two-dof vibration absorber, please refer to reference [4].

V. Optimal Parameters for the Two-dof Vibration Absorber Attached to a Beam

The non-dimensional optimal absorber parameters presented in the last section are for the case of the absorber to be attached to a generalized main vibration system, as shown in Figure 2(a). If the absorber is attached to a multiple dof beam (cf. Figure 5), the non-dimensional optimal parameters for the absorber may be determined by the technique presented in this section. Once the non-dimensional optimal parameters for the two-dof vibration absorber are obtained, the corresponding dimensional optimal ones, such as spring constant k_a and damping coefficient c_a of the two-dof vibration absorber, can be calculated.

A. Non-dimensional optimal parameters for the two-dof vibration absorber attached to a beam

The equation of motion of the undamped beam without carrying absorber takes the form [15]

$$[M]_{n'\times n'}\{\ddot{q}(t)\}_{n'\times 1}+[K]_{n'\times n'}\{q(t)\}_{n'\times 1}=\{F(t)\}_{n'\times 1} \quad (48)$$

where $[M]$ and $[K]$ are respectively the effective overall mass and stiffness matrices, $\{\ddot{q}(t)\}$, $\{\dot{q}(t)\}$ and $\{q(t)\}$ are respectively the acceleration, velocity and displacement vectors, $\{F(t)\}$ is the external force vector and n' is the effective total degree of freedom of the beam.

By applying the theory of mode superposition method and the orthogonal property of the normal mode shapes of the beam to Equation (48), the n' coupled simultaneous differential equations can be reduced to the following uncoupled ones [13]

$$\{\phi_r\}^T[M]\{\phi_r\}\ddot{\eta}_r+\{\phi_r\}^T[K]\{\phi_r\}\eta_r=\{\phi_r\}^T\{F(t)\} \quad (49)$$

or

$$\tilde{m}_r\ddot{\eta}_r+\tilde{k}_r\eta_r=\tilde{f}_r \ \ (r=1-n') \quad (50)$$

where $\tilde{m}_r$, $\tilde{k}_r$ and $\tilde{f}_r$ are respectively the generalized mass, generalized stiffness and generalized force for the generalized main vibration system associated with the r-th vibration mode of the beam, $\ddot{\eta}_r$ and η_r are respectively the r-th generalized acceleration and displacement, while $\{\phi_r\}$ is the r-th normal mode shape.

For a beam subjected to an eccentric moving force, as shown in Figure 3(a), the contribution to the torsional-vibration responses of the beam from its first torsional-vibration mode (i.e., $r = n_p$) is the most significant, where n_p is the mode number for the first torsional-vibration mode of the beam. Therefore, one may design an optimal absorber according to the generalized mass and generalized stiffness of the generalized main vibration system associated with the n_p^{th} vibration mode of the beam, $\tilde{m}_{n_p}$ and $\tilde{k}_{n_p}$. It is evident that the values of $\tilde{m}_{n_p}$ and $\tilde{k}_{n_p}$ may be obtained from Equation (49) with $r = n_p$. If the mass moment of inertia of the absorber, J_a, is given, then the associate non-dimensional optimal parameters for the two-dof vibration absorber may be obtained from Equations (45)-(47).

B. Dimensional optimal parameters for the two-dof vibration absorber attached to a beam

The non-dimensional optimal parameters of the two-dof vibration absorber for suppressing the torsional-vibration responses of the beam are presented in the last section. They are found to be the functions of mass ratio $\mu = J_a / \bar{J}_{\theta x}$ and are given by

$$f_{opt} = \frac{\sqrt{2k_a d^2 / J_a}}{\sqrt{\bar{k}_{\theta x} / \bar{J}_{\theta x}}} = \frac{1}{1+\mu} \quad (51)$$

$$\xi_{opt} = \frac{2c_a d^2}{2\sqrt{2J_a k_a d^2}} = \sqrt{\frac{3\mu}{8(1+\mu)}} \quad (52)$$

where the last two equations are obtained from Equations (42), (43), (46) and (47).

From Equations (51) and (52), one obtains

$$k_a = \frac{\mu \bar{k}_{\theta x}}{2d^2} f_{opt}^2 \quad (53)$$

$$c_a = \frac{\sqrt{2J_a k_a}}{d} \xi_{opt} \quad (54)$$

Equations (53) and (54) are the expressions for calculating the dimensional optimal spring constant k_a and damping coefficient c_a of the two-dof vibration absorber.

VI. Dynamic Responses of a Beam with and without Carrying a Two-dof Vibration Absorber Undergoing an Eccentric Moving Force

If, at any instant of time t, the locus of the concentrated force P is not coincident with the centreline of the beam, then the last eccentric concentrated force P (see Figure 3(a)) can be replaced by an equivalent concentrated force $\bar{P} = P$ together with a moment $\bar{M} = P\bar{e}$ (see Figure 3(b)). Where $\bar{e}$ is the distance between the concentrated force P and the centreline of the beam. In other words, if the locus of the concentrated force P is coincident with the centreline of the beam, then $\bar{e} = 0$. Thus, $\bar{P} = P$ and $\bar{M} = 0$. The aforementioned concept has been investigated by Wu [16] and satisfactory results have been obtained.

For a multiple-degree-of-freedom beam, its equation of motion is given by [15]

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{F(t)\} \quad (55)$$

where $[M]$, $[C]$ and $[K]$ are the overall mass, damping and stiffness matrices of the *unconstrained* beam, respectively, $\{\ddot{u}(t)\}$, $\{\dot{u}(t)\}$ and $\{u(t)\}$ are the acceleration, velocity and displacement vectors of the entire structural system, respectively, and $\{F(t)\}$ is the overall external force vector.

For convenience, in this paper, a beam carrying any number of absorbers is called the *loaded beam*, while that carrying nothing is called the *bare beam*.

A. Dynamic responses of a beam without carrying an absorber undergoing an eccentric moving force

In Equation (55), the overall stiffness and mass matrices, $[K]$ and $[M]$, of the bare beam can be obtained by assembling the elementary properties matrices of each beam element [17]. Because the beam studied herein is undamped, the overall damping matrix of the beam is a null matrix, i.e.,

$$[C] = [0] \quad (56)$$

Based on the conventional finite element method, if a beam is subjected to a concentrated force $\bar{P}$ and moment $\bar{M}$, all the nodal forces of the beam are equal to zero except those for the s^{th} beam element on which the concentrated force $\bar{P}$ and moment $\bar{M}$ apply (see Figure 4). For this reason, the overall external force vector $\{F(t)\}$ in Equation (55) takes the form

$$\{F(t)\} = [0\,0\,0 \cdots f_1^{(s)}(t)\ f_2^{(s)}(t)\ f_3^{(s)}(t)\ f_4^{(s)}(t)\ f_5^{(s)}(t)\ f_6^{(s)}(t) \cdots 0\,0\,0]^T \quad (57)$$

where $f_i^{(s)}(t)$ (i = 1 to 6) are the equivalent nodal forces of the s^{th} beam element on which the moving force $\bar{P}$ and moment $\bar{M}$ apply and are given by [16]

$$\{f^{(s)}(t)\} = [f_1^{(s)}(t)\quad f_2^{(s)}(t)\quad f_3^{(s)}(t)\quad f_4^{(s)}(t)\quad f_5^{(s)}(t)\quad f_6^{(s)}(t)]^T = [\bar{P}N_1\quad \bar{P}N_2\quad \bar{M}N_3\quad \bar{P}N_4\quad \bar{P}N_5\quad \bar{M}N_6]^T \quad (58)$$

where N_i (i =1 to 6) are the shape functions of the beam element given by [15]

$$N_1 = 1 - 3\varsigma^2 + 2\varsigma^3 \quad (59)$$

$$N_2 = (\varsigma - 2\varsigma^2 + \varsigma^3)L_b \quad (60)$$

$$N_3 = 1 - \varsigma \quad (61)$$

$$N_4 = 3\varsigma^2 - 2\varsigma^3 \quad (62)$$

$$N_5 = (-\varsigma^2 + \varsigma^3)L_b \quad (63)$$

$$N_6 = \varsigma \quad (64)$$

$$\varsigma = x / L_b \quad (65)$$

where L_b and x respectively represent the length of the beam element and the distance between the position of the concentrated force $\bar{P}$ (or moment $\bar{M}$) and the left-end of the beam element (see Figure 4).

It is worthy of mention that, in Equation (57), the equivalent nodal forces, $f_i^{(s)}(t)$ (i = 1 to 6), are the s_i^{th} (i = 1 to 6) coefficients of $\{F(t)\}$, respectively, where s_i (i = 1 to 6) are the numberings for the six degrees of freedom of the s^{th} beam element on which the moving force (or moment) applies.

If the concentrated force $\bar{P}$ (or moment $\bar{M}$) moves, with a constant velocity V, from the left-end to the right-end of the beam, the position of the concentrated force $\bar{P}$ (or moment $\bar{M}$) at any instant of time t is given by

$$\bar{x}_p(t) = Vt \tag{66}$$

Thus, the numbering for the beam element on which the concentrated force $\bar{P}$ (or moment $\bar{M}$) applies at time t is

$$s = (\text{ Integer part of } \frac{\bar{x}_p(t)}{L_b}) + 1 \tag{67}$$

In such a case, the instantaneous overall external force vector $\{F(t)\}$, as shown in Equation (55), can be determined by using Equations (57)-(67). It is noted that the local x coordinate of the moving concentrated force $\bar{P}$ (or moment $\bar{M}$), as shown in Equation (65), is a function of the global coordinate $\bar{x}_p(t)$, i.e.,

$$\varsigma = \frac{x}{L_b} = \frac{\bar{x}_p(t) - (s-1)L_b}{L_b} \tag{68}$$

Imposing the prescribed boundary conditions of the beam to Equation (55) gives the equation of motion of the *constrained* beam, i.e.,

$$[\bar{M}]\{\ddot{\bar{u}}(t)\} + [\bar{C}]\{\dot{\bar{u}}(t)\} + [\bar{K}]\{\bar{u}(t)\} = \{\bar{F}(t)\} \tag{69}$$

where $[\bar{M}]$, $[\bar{C}]$ and $[\bar{K}]$ are the effective overall mass, damping and stiffness matrices, respectively, and $\{\bar{F}(t)\}$ is the effective overall external force vector of the *constrained* beam.

Finally, the dynamic responses of the bare beam undergoing an eccentric moving force can be determined by solving the equation of motion of the constrained beam, Equation (69), by using Newmark direct integration method [18].

B. Dynamic responses of a beam carrying a two-dof vibration absorber undergoing an eccentric moving force

The formulations of the last section are available for calculating the dynamic responses of the bare beam undergoing an eccentric moving force. To calculate the dynamic responses of the loaded beam (see Figure 5) undergoing an eccentric moving force, the overall mass, damping and stiffness matrices, $[M]$, $[C]$ and $[K]$, of the last section must be replaced by

$$[\tilde{M}]_{(n+2)\times(n+2)} = [M]_{n\times n} + [m_a]_{4\times 4} \tag{70}$$

$$[\tilde{C}]_{(n+2)\times(n+2)} = [C]_{n\times n} + [c_a]_{4\times 4} \tag{71}$$

$$[\tilde{K}]_{(n+2)\times(n+2)} = [K]_{n\times n} + [k_a]_{4\times 4} \tag{72}$$

where n represents the total degree of freedom of the bare beam, while $[m_a]$, $[c_a]$ and $[k_a]$ represent the stiffness, damping and mass matrices of the two-dof vibration absorber given by Equations (22), (23) and (24), respectively. From the last equations, one sees that the total degree of freedom of the bare beam carrying a two-dof vibration absorber is two more than that of the bare beam without carrying an absorber. It is noted that the assembly of the element property matrices of the absorber, $[m_a]$, $[c_a]$ and $[k_a]$, and the mass, damping and stiffness matrices, $[M]$, $[C]$ and $[K]$ of the bare beam must be conducted according to the numberings for the degrees-of-freedom of the absorber. The effective overall property matrices of the *constrained* beam carrying an absorber, $[\bar{M}]$, $[\bar{C}]$ and $[\bar{K}]$, can be obtained from the overall matrices $[\tilde{M}]$, $[\tilde{C}]$ and $[\tilde{K}]$ by imposing the prescribed boundary conditions.

Now, one can determine the dynamic responses of the beam carrying a two-dof vibration absorber undergoing an eccentric moving force by using the similar procedures of the last subsection

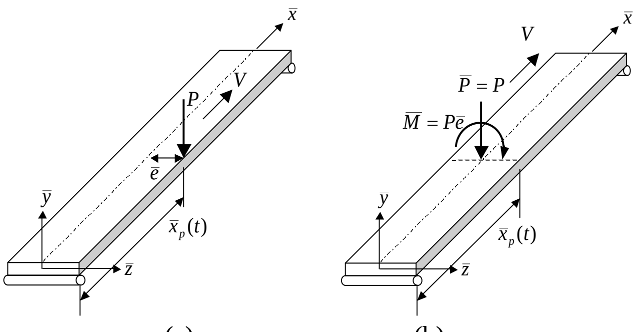

Figure 3 (a) A pinned-pinned beam subjected to an eccentric moving force, with magnitude P and eccentricity $\bar{e}$, can be replaced by (b) the same beam subjected to an equivalent force $\bar{P} = P$ together with an equivalent moment $\bar{M} = P\bar{e}$ moving along the centerline of the beam.

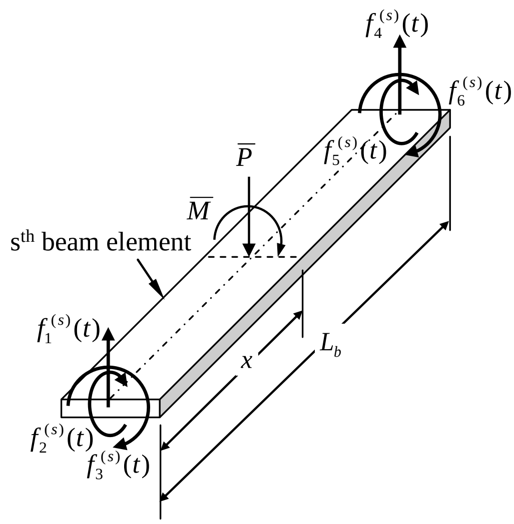

Figure 4 Equivalent nodal forces $f_i^{(s)}$ (i =1 to 6) of the s^{th} beam element due to a concentrated force $\bar{P}$ and a concentrated moment $\bar{M}$.

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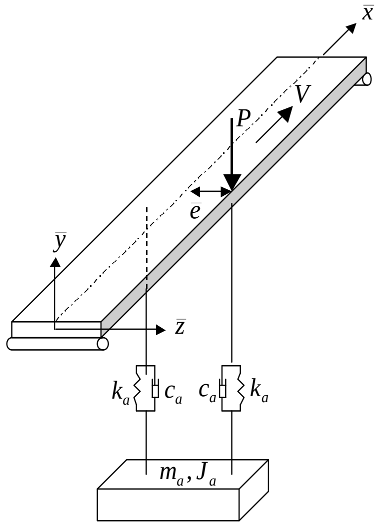

Figure 5 A pinned-pinned beam carrying a two-dof vibration absorber subjected to an eccentric moving force, with magnitude P and eccentricity $\bar{e}$, moving from the left end to the right end of the beam with a constant speed V.

VII. Numerical Results and Discussions

To confirm the reliability of the presented theory and the developed computer programs, numerical investigations on the optimal parameters of the two-dof vibration absorber for suppressing the rotational-vibration responses of a generalized main vibration system due to a harmonic torque are conducted first. Then, the last optimal parameters of the two-dof vibration absorber are applied to the vibration reduction of the torsional-vibration responses of a uniform undamped pinned-pinned beam induced by an eccentric moving force.

A. Influence of frequency ratio

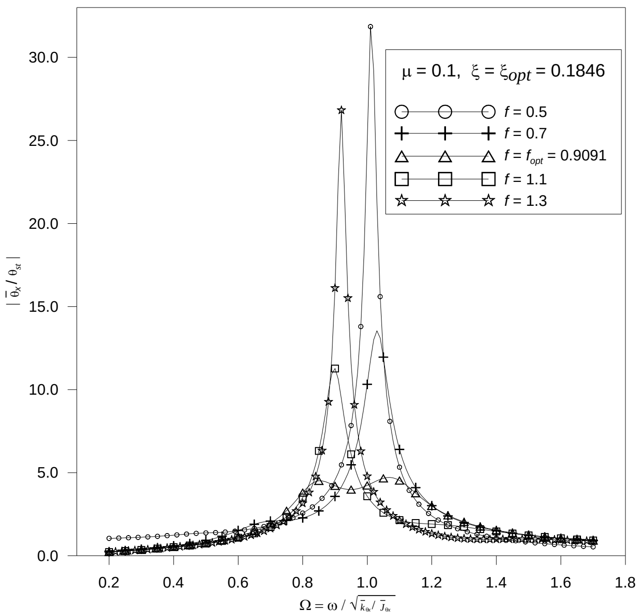

Figure 6 Influence of frequency ratio (f) of the two-dof vibration absorber on the dynamic magnification factor $\left|\bar{\theta}_x/\theta_{st}\right|$ of the generalized main vibration system (see Figure 2(a)).

The example studied here is a generalized main vibration system carrying a two-dof vibration absorber, as shown in Figure 2(a). Based on the ratio for mass moment of inertia $\mu = J_a/\bar{J}_{\theta x}$ = 0.1 together with the optimal damping ratio $\xi = \xi_{opt}$ = 0.1846 and frequency ratios f = 0.5, 0.7, 0.9091, 1.1 and 1.3, the curves for the dynamic magnification factor of the rotational angles (θ_x) of the generalized main vibration system, $\left|\bar{\theta}_x/\theta_{st}\right|$, versus frequency ratio, $\Omega = \omega\Big/\sqrt{\bar{k}_{\theta x}/\bar{J}_{\theta x}}$, are obtained from Equation (39) and shown in Figure 6. It is noted that the optimal damping ratio ($\xi = \xi_{opt}$ = 0.1846) and optimal frequency ratio ($f = f_{opt}$ = 0.9091) are obtained from Equations (47) and (46), respectively. In Figure 6, the solid curves with circles, crosses, triangles, rectangles and stars, —○—, —+—, —Δ—, —□— and —☆—, are for the cases with frequency ratios of the absorber f = 0.5, 0.7, 0.9091, 1.1 and 1.3, respectively. It is evident that, among the last five curves, the smallest maximum dynamic magnification factor occurs when $f = f_{opt}$ = 0.9091.

B. Influence of damping ratio

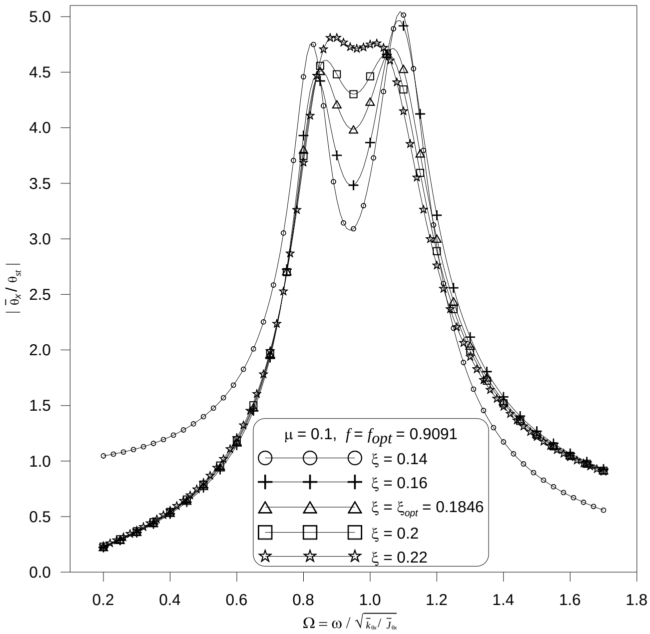

Figure 7 Influence of damping ratio (ξ) of the two-dof vibration absorber on the dynamic magnification factor $\left|\bar{\theta}_x/\theta_{st}\right|$ of the generalized main vibration system (see Figure 2(a)).

All the parameters for the case studied in this subsection are exactly the same as those of the last subsection except that the frequency ratio of the two-dof vibration absorber is $f = f_{opt}$ = 0.9091 and the damping ratios are ξ = 0.14, 0.16, 0.1846, 0.20 and 0.22. Clearly, $f = f_{opt}$ = 0.9091 and $\xi = \xi_{opt}$ = 0.1846 are the optimal parameters of the two-dof vibration absorber obtained from Equations (46) and (47), respectively. Figure 7 shows the curves for dynamic magnification factor of the generalized main vibration system, $\left|\bar{\theta}_x/\theta_{st}\right|$, versus frequency ratio, $\Omega = \omega\Big/\sqrt{\bar{k}_{\theta x}/\bar{J}_{\theta x}}$. In which, the solid curves with circles, crosses, triangles, rectangles and stars, —○—, —+—, —Δ—, —□— and —☆—, are for the cases with damping ratios ξ = 0.14, 0.16, 0.1846, 0.20 and 0.22, respectively. From the figure, one finds that, among the curves, the smallest maximum dynamic magnification factor occurs when $\xi = \xi_{opt}$ = 0.1846.

From the numerical results of this subsection and the last one, one sees that the vibration-reduction efficiency of the two-dof vibration absorber will be significantly increased if the frequency ratio and damping ratio of the two-dof vibration absorber approach their optimal values, respectively. Therefore, one believes that the presented optimal parameters of the two-dof vibration absorber do provide an effective technique for suppressing the rotational-vibration responses of the generalized main vibration system.

C. Influence of the ratio for mass moment of inertia

Table 1 Optimal frequency ratio and damping ratio of the two-dof vibration absorber corresponding to $\mu = J_a / \bar{J}_{\theta x}$ = 0.1, 0.2, 0.3, 0.4 and 0.5.

Ratios for mass moment of inertia $\mu = J_a / \bar{J}_{\theta x}$	Optimal frequency ratios f_{opt}	Optimal damping ratios ξ_{opt}
0.1	0.9091	0.1846
0.2	0.8333	0.2500
0.3	0.7692	0.2942
0.4	0.7143	0.3273
0.5	0.6667	0.3536

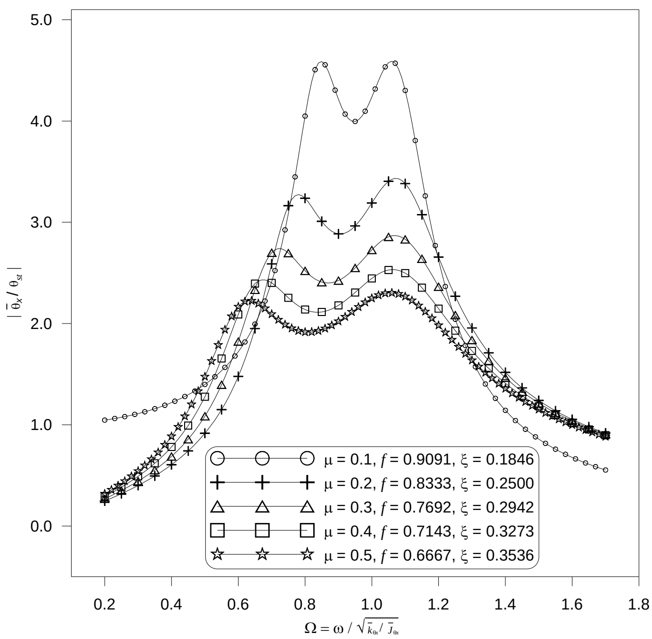

Figure 8 Influence of the ratio for mass moment of inertia (μ) of the two-dof vibration absorber on the dynamic magnification factor $\left|\bar{\theta}_x / \theta_{st}\right|$ of the generalized main vibration system (see Figure 2(a)).

The differences between the current example and the last one are that the ratios for mass moment of inertia are $\mu = J_a / \bar{J}_{\theta x}$ = 0.1, 0.2, 0.3, 0.4 and 0.5. Besides, the optimal frequency ratio and optimal damping ratio of the two-dof vibration absorber corresponding to each ratio for mass moment of inertia (μ) are listed in Table 1. In the table, the optimal frequency ratio and damping ratio listed in second and third columns are calculated using the ratios for mass moment of inertia listed in the first column and Equations (46) and (47), respectively. Figure 8 shows the curves for dynamic magnification factor of the generalized main vibration system, $\left|\bar{\theta}_x / \theta_{st}\right|$, versus frequency ratio, $\Omega = \omega / \sqrt{\bar{k}_{\theta x} / \bar{J}_{\theta x}}$. In the figure, the solid curves with circles, crosses, triangles, rectangles and stars, —○—, —+—, —Δ—, —□— and —☆—, are for the cases with μ = 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. From the figure, it is seen that the larger the ratio for mass moment of inertia, the better the vibration-reduction efficiency of the two-dof vibration absorber. This is a reasonable result because the two-dof vibration absorber can dissipate more energy if the mass moment of inertia of the absorber is increased.

D. Design of a two-dof vibration absorber for suppressing the torsional-vibration responses of a beam due to an eccentric moving force

To show the applicability of the presented theory, the optimal parameters of a two-dof vibration absorber for suppressing the torsional-vibration responses of a pinned-pinned beam subjected to an eccentric moving force, with magnitude P = 49 N and eccentricity $\bar{e}$ =0.175 m, moving from the left-end to the right-end of the beam (see Figures 3(a) and 5) with a constant speed V, are investigated. The beam, composed of 13 nodes and 12 identical beam elements, is made of steel with mass density ρ = 7820 kg/m^3, Young's modulus E = 206.8 GN/m^2, total length L = 4 m and cross sectional area A = 0.35 $m \times$ 0.03 m. Note that the two-dof vibration absorber is installed to the central point of the beam (i.e., the crest of the mode shape corresponding to the first torsional-vibration mode of the beam).

For a pinned-pinned beam subjected to an eccentric moving force, the contribution to the torsional-vibration responses of the beam from its first torsional-vibration mode is the most significant. Therefore, this section will design an optimal two-dof vibration absorber according to the modal data of the first torsional-vibration mode of the beam. To this end, an absorber will be installed to the central point of the beam (i.e., the crest for the mode shape of the first torsional-vibration mode of the beam), as shown in Figure 5. According to Equations (49) and (50), the generalised mass $\tilde{m}_1$ and generalised stiffness $\tilde{k}_1$ associated with the first torsional-vibration mode of the beam are respectively given by $\tilde{m}_1$ = 1.6696 $kg.m^2$ and $\tilde{k}_1$ = 292993.0 Nm/rad. Thus, the mass moment of inertia and rotational spring constant for the generalized main vibration system, simplified from the pinned-pinned beam, are $\bar{J}_{\theta x} = \tilde{m}_1$ = 1.6696 $kg.m^2$ and $\bar{k}_{\theta x} = \tilde{k}_1$ = 292993.0 Nm/rad.

In general, the larger the mass moment of inertia of the two-dof vibration absorber, the better the vibration-reduction efficiency of the absorber. From Equation (25), one sees that the mass moment of inertia of the absorber J_a is proportional to the magnitude of its lumped mass m_a. However, the larger the lumped mass of the absorber, the larger the static deflection of the beam. Thus, the lumped mass of the absorber, m_a, cannot be too large. In this subsection, the lumped mass of the absorber is taken to be 5% of the total mass of the pinned-pinned beam, i.e., $m_a = 7820 \times 0.35 \times 0.03 \times 4 \times 0.05 =$ 16.422 *kg*. Thus, mass moment of inertia of the absorber is $J_a = \frac{1}{12} m_a (\ell_a^2 + \ell_b^2) = \frac{1}{12} \times 16.422 \times (0.35^2 + 0.3^2) = 0.2908$ *kg.m*^{2}, where ℓ_a and ℓ_b are respectively the length and height of the lumped mass m_a (cf. Equation (25) and Figure 2(b)). Based on the mass moment of inertia $\bar{J}_{\theta x}$ = 1.6696 *kg.m*^{2}, rotational spring constant $\bar{k}_{\theta x}$ = 292993.0 *Nm/rad*, mass moment of inertia of the absorber $J_a = 0.2908$ *kg.m*^{2} and Equations (45)-(47), one obtains the ratio for mass moment of inertia $\mu = J_a / \bar{J}_{\theta x}$ =0.1741 and its corresponding non-dimensional optimal frequency ratio f_{opt}=0.8517 and non-dimensional optimal damping ratio ξ_{opt}=0.2358. If the distance $d = \ell_a / 2$ =0.175*m*, from Equations (53) and (54), the dimensional optimal spring constant and damping coefficient are found to be k_a =604120.005 *Nm/rad* and c_a =798.692 *Nms/rad*, respectively.

The maximum torsional-vibration responses (i.e., rotational angles about the $\bar{x}$ axis) of the central point of the pinned-pinned beam, with and without a two-dof vibration absorber, subjected to an eccentric moving force (with magnitude $P = 49$ *N*, eccentricity $\bar{e}$ =0.175 *m* and constant speed V =1 to 50 m/sec) are shown in Figure 9. From the figure, one finds that the two-dof vibration absorber does suppress the torsional-vibration responses of the pinned-pinned beam subjected to an eccentric moving force. Figure 10 shows the maximum flexural-vibration responses ($\bar{y}$) of the central point of the pinned-pinned beam. From the figure, it can be found that the two-dof vibration absorber can also suppress the flexural -vibration responses ($\bar{y}$) of the beam to some degree.

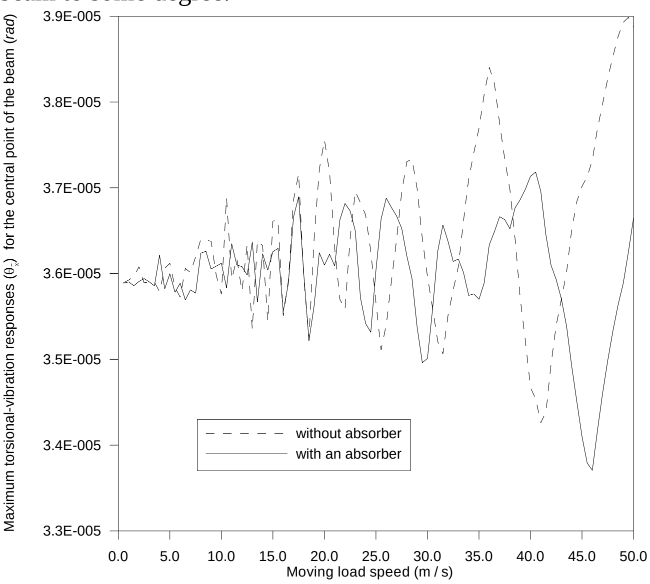

Figure 9 Maximum torsional-vibration responses ($\theta_{\bar{x}}$) for the central point of the beam with and without a two-dof vibration absorber (m_a =16.422 *kg*, $J_a = 0.2908$ *kg.m*^{2}, d =0.175*m*, k_a =604120.005 *N/m* and c_a =798.692 *Ns/m*).

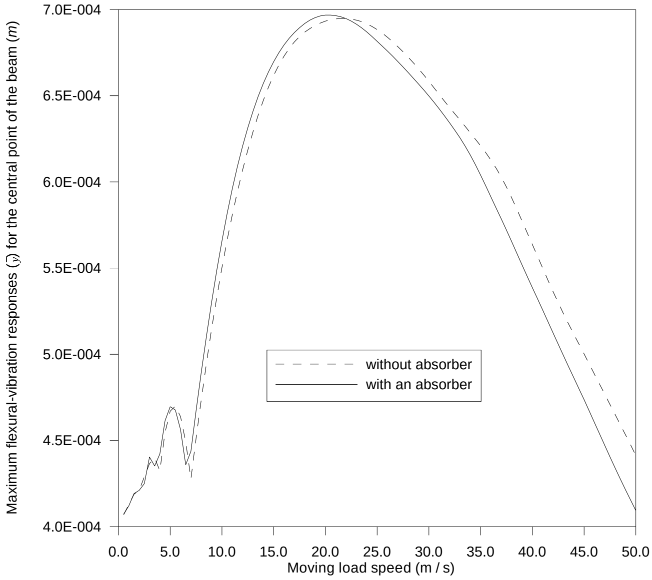

Figure 10 Maximum flexural-vibration responses ($\bar{y}$) for the central point of the beam with and without a two-dof vibration absorber (m_a =16.422 *kg*, $J_a = 0.2908$ *kg.m*^{2}, d =0.175*m*, k_a =604120.005 *N/m* and c_a =798.692 *Ns/m*).

VIII. CONCLUSION

This paper presents a technique for determining the optimal parameters of a two-dof vibration absorber, so that the torsional-vibration responses of a structure induced by external dynamic loads can be effectively suppressed. Numerical results reveal that the presented optimal parameters of a two-dof vibration absorber can effectively suppress the torsional-vibration responses of structures. In view of the fact that the presented two-dof vibration absorber can also suppress the translational structural responses to some degree, the presented two-dof vibration absorber should be better than the conventional one-dof vibration absorber.

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