# Availability of a Redundant System with Two Parallel Active Components under Markovian Assumptions

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*Abstract*—**We consider a redundant system which consists of two parallel active components. The time-to-failure and the time-to-repair of the components follow exponential distributions. The repairs of failed components are randomly interrupted. The time-to-interrupt is taken from an exponentially distributed random variable and the interrupt times are also exponentially distributed. We obtain the availability for the system.**

*Index Terms*— **Availability, Markovian, parallel system, redundancy**

## I. INTRODUCTION

Availability is defined as the probability that a system is operational at a given point in time under a given set of environmental conditions. There have been efforts to improve the availability. Redundant systems are typically used to improve the availability. There are various redundant systems to appropriately support uptime requirements in the industry.

The availability analysis of a system is based on analyzing the various states that the system undergoes during its lifespan. Since the occurrence of failures is erratic by nature, stochastic models have been used to conduct the availability analysis. Markov models have been extensively used, because of their expressiveness and their capability of capturing the complexity of real systems [1]-[4].

The most existing literature has focused on uninterrupted repairs with exponentially distributed repair time. Kuo and Ke [5] studied the availability of a series system with interrupted repairs and generally distributed repair time. Bosse et al. [6] estimated the availability of a redundant system with imperfect switchovers and interrupted repairs by using a Petri net Monte Carlo simulation. Lee [7] analyzed the availability of a system with one active and one standby component. In this paper, we focus on the availability for a parallel redundant system with two active components and interrupted repairs.

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## II. MODEL

We consider a redundant system with two parallel active components. We assume that each component fails independently of the state of the other. Let the time-to-failure of the active components be exponentially distributed with rate $\alpha$. The repair time is also exponentially distributed with $\beta$. Moreover, the repairer may function wrongly or fail sometimes in its busy period with an exponential failure rate $\gamma$. Once the repairer becomes available again, it resumes the interrupted process. The interrupted time is also exponentially distributed with rate $\delta$.

For any $t \geq 0$, define $S(t)$ as the state of the system at time $t$: the value $S(t)$ is $0$ if the two components are failed and their repair is interrupted; the value $S(t)$ is $1$ if two components are failed and one of them is being repaired; the value $S(t)$ is $2$ if one component is active, the other is failed, and the repair of the failed one is interrupted; the value $S(t)$ is $3$ if one component is active, the other is failed, and the failed one is being repaired; The value $S(t)$ is $4$ if the two components are active.

The sojourn time in each state is exponentially distributed. Thus, the stochastic process $\{S(t)\}$ is a continuous-time Markov chain with state space $\{0,1,2,3,4\}$.

## III. AVAILABILITY

The steady-state behavior of the system is examined in this section. Let:

$$p_i \equiv \lim_{t \to \infty} P\{S(t) = i\} \tag{1}$$

The balance equations governing the system are:

$$\delta p_0 = \gamma p_1 + \alpha p_2 \tag{2}$$

$$(\beta + \gamma)p_1 = \delta p_0 + \alpha p_3 \tag{3}$$

$$(\alpha + \delta)p_2 = \gamma p_3 \tag{4}$$

$$(\alpha + \beta + \gamma)p_3 = \beta p_1 + \delta p_2 + 2\alpha p_4 \tag{5}$$

$$2\alpha p_4 = \beta p_3 \tag{6}$$

After some calculations, we obtain:

$$p_0 = \frac{1}{\delta}\left[\gamma \frac{1}{\beta}\left(\alpha + \gamma - \delta \frac{\gamma}{\alpha + \delta}\right) + \alpha \frac{\gamma}{\alpha + \delta}\right]p_3 \tag{7}$$

$$p_1 = \frac{1}{\beta}\left[\alpha + \gamma - \delta \frac{\gamma}{\alpha + \delta}\right]p_3 \tag{8}$$

$$p_2 = \frac{\gamma}{\alpha + \delta}p_3 \tag{9}$$

$$p_4 = \frac{\beta}{2\alpha}p_3 \tag{10}$$

From the normalization condition:

$$p_0 + p_1 + p_2 + p_3 + p_4 = 1 \tag{11}$$

we have:

$$p_0 = \frac{\frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(12)

$$p_1 = \frac{\frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right]}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(13)

$$p_2 = \frac{\frac{\gamma}{\alpha + \delta}}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(14)

$$p_3 = \frac{1}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(15)

$$p_4 = \frac{\frac{\beta}{2\alpha}}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(16)

The availability $Av$ can be obtained as:

$$Av = p_2 + p_3 + p_4$$

$$= \frac{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta}}{1 + \frac{\beta}{2\alpha} + \frac{\gamma}{\alpha + \delta} + \frac{1}{\beta}\left[\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right] + \frac{1}{\delta}\left[\gamma\frac{1}{\beta}\left(\alpha + \gamma - \delta\frac{\gamma}{\alpha + \delta}\right) + \alpha\frac{\gamma}{\alpha + \delta}\right]}$$

(17)

## IV. CONCLUSION

This paper has obtained the analytical expression of the steady-state availability for a redundancy model with two parallel active components. The time-to-failure and the time-to-repair of the components follow exponential distributions. The repairs of failed components are randomly interrupted. The time-to-interrupt is taken from an exponentially distributed random variable and the interrupt times are also exponentially distributed. This paper has obtained the availability for the system.

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