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Fuzzy Norm Method for Evaluation of Uncertainty in Vibration Performance of Rolling Bearing

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***Abstract*— With the rapid development of science and technology, the performance of current bearings in terms of variations in friction, wear, vibration, temperature rise, etc. is drawing considerable attention. The limited availability of characterization data and a lack of a priori knowledge concerning the probability distribution and trends in bearing performance have rendered it difficult to perform a statistical analysis on this topic. To overcome these limitations, in this paper, we adopt the fuzzy norm method, which integrates fuzzy theory with the norm method. We utilize the fuzzy norm method to assess the uncertainty in rolling bearing vibration performance, which can help to reveal the degree of variation in rolling bearing vibration performance despite the unknown probability distribution and trends, thereby allowing for accurate assessment of the vibration conditions of bearings. The results of an experimental investigation of a specific type of rolling bearing vibration-acceleration time series demonstrate the correctness and effectiveness of the proposed method.**

***Index Terms*—Rolling bearings, vibration performance, uncertainty, fuzzy theory, norm method.**

I. INTRODUCTION

A rolling bearing is a key component for the proper operation of its host system. With developments in science and technology, new requirements have been placed on the performance variability of rolling bearings in terms of friction, wear, vibration, and temperature rise, among other factors. The performance reliability of a rolling bearing, such as its rotational accuracy, vibrations, friction torque fluctuations and lifetime, can directly affect the working performance and service life of its host [1-3].

Rolling bearings are widely used in many hosts, such as CNC machine tools, submarines, automobiles, bullet trains, aircrafts, and spacecrafts. In the late 90's of 20th century, with the continuous enhancement of host performance in many enterprises, especially in light car production enterprises, bullet train production enterprises, and CNC machine tool production enterprises, which take small, small and medium-sized rolling bearings as important components of hosts, higher requirements for the quality of rolling bearing vibration have been put forward, so that reliability and stability of the running condition of these hosts can be improved significantly. Obviously, the level of rolling bearing vibration performance has become the bottleneck of restricting the advancement of overall quality level of these hosts. It follows that more attention must focus on study of rolling bearing vibration performance. From this point of view, the monitoring and evaluation of variations in rolling bearing performance is of great significance in engineering practice.

Damage and wear of internal parts can cause bearing vibration, and conversely, bearing vibration may increase the susceptibility of the internal parts to damage and wear. A continuous cycle of this process may result in "vibration-induced damage to the bearing" and may cause variations in the bearing's vibration performance.

At present, experimental investigations of rolling bearing vibration performance have received limited research interest, and most related studies have primarily focused on influence factor analysis and fault diagnosis [4-11]. For example, an experimental study and comparative analysis of the vibration performances of steel ball bearings with or without CrC coatings has shown that the application of CrC coating to a steel ball bearing significantly reduces the vibration of the bearing, especially under high-speed and high-load conditions, thus improving the performance stability in terms of bearing vibration [6]. Another method for the dynamic assessment and diagnosis of rolling bearing vibration, known as the grey bootstrap method, yields an evaluation of the fundamental dynamic characteristics of the vibration of a rolling bearing based on 6 evaluation parameters. According to the evaluation results, the model also reveals the effect of the nature of the error on the bearing vibration and then lays a foundation for implementing the production process control of the rolling bearing vibration by revealing the source of the error in the manufacturing of the bearing parts [7]. Meanwhile, fault diagnosis strategies based on the relevance vector machine (RVM) approach and kernel principal component analysis methods improve the speed and accuracy of rolling bearing fault diagnosis in the case of small samples [8]. Moreover, the autoregressive model of modern spectral estimation can overcome the low resolution and poor variance of the classical power spectrum to enable the fault diagnosis of rolling bearing vibrations in the presence of noise and improve the diagnostic speed [9].

Measurement uncertainties can give rise to fluctuations in the range of values measured for a certain quantity with a given true value. The dynamic measurement uncertainty in bearing vibration values can be used to characterize the degree of variation in vibration performance. At present, only a few studies have researched methods of evaluating this dynamic measurement uncertainty [12-14]. For example, in the dynamic linear model with constant mean, the Bayesian uncertainty is first estimated; then, the components of the dynamic linear model are analysed, and the dynamic measurement uncertainty is evaluated by modelling the ergodic process [13]. In view of the shortcomings of slow convergence speed and unstable simulation results

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encountered in Monte Carlo methods, a method of dynamic measurement uncertainty evaluation based on a quasi-Monte Carlo method has been proposed; this study began with the characteristics of a dynamic measurement system, and through the introduction of a low bias point, the authors were able to generate a quasi-random number sequence with a more uniform spatial distribution instead of a pseudo-random number sequence of the type used in Monte Carlo methods [14]. However, in these investigations, the uncertainty in the rolling bearing vibration performance was not taken into account, and therefore, in this study, a new, simple and economical method based on measured data collected by an acceleration sensor is presented to recognize and evaluate the vibration conditions of a rolling bearing.

Because of the limited availability of characterization data and the lack of a priori knowledge concerning the probability distribution and trends in rolling bearing vibration performance, it is difficult to perform a statistical analysis on this topic. Therefore, the fuzzy norm method, which integrates fuzzy theory with the norm method, is adopted for the assessment of the uncertainty in rolling bearing vibration performance [15-18]. For example, a new method adopts a fuzzy practicable interval to characterize non-statistical uncertainty in dynamic measurement, and the method permits the uncertainty being estimated under the conditions that the number of measurements is very small and the probability distribution unknown. The feasibility of the method is validated using computer-simulation experiments [16]. In fuzzy time series analysis, the determination of the interval length is an important issue. The length of intervals has been intuitively determined in many researches recently done. The authors propose a new method based on the use of a single variable constrained optimization to determine the length of interval [17].

In order to make a further research on rolling bearing vibration performance, a novel research idea using the fuzzy method is proposed, and the fuzzy norm method can help to reveal the degree of variation in rolling bearing vibration performance given an unknown probability distribution and unknown trends, thereby allowing for the accurate assessment of the vibration conditions of bearings.

II. Mathematical Model

A. The fuzzy practicable interval of measured data

The rolling bearing vibration performance under investigation is represented by a random variable x. During rolling bearing operation or experiments, the vibration performance is assumed to be regularly subjected to sample analysis through the acquisition of performance data for R time units. X_r represents the measured data for the rth time unit, and the rth time series is expressed as

$$X_r = \{x_r(k)\};\ k = 1, 2, \ldots, n;\ r = 1, 2, \ldots, R \tag{1}$$

where $x_r(k)$ represents the kth original data recorded, k is the number of current data points in X_r, n is the number of data points in X_r, r is the number of current time unit, and R is the number of time units.

Fuzzy mathematics using membership functions is applied to investigate the intermediate transition laws of a fuzzy entity with a status that is changing from true to false or from false to true. The measured true value X_0 is objective and unique. Thus, the set A is defined as

$$A = \{X_0\} \tag{2}$$

where X_0 is a single value.

On the basis of set theory, the measured values x_i (i=1, 2,…, n) and the set A satisfy the following binary-valued logic characteristic function $G_A(x)$:

$$G_A(x) = \begin{cases} 1 & x_i \in A \\ 0 & x_i \notin A \end{cases} \tag{3}$$

where 1 indicates true and 0 indicates false; n is the total number of measurements.

In fuzzy-set theory, the membership function of x_i with respect to A indicates the degree of approximation from x_i to A, which can be regarded as a transition, and the interval B of this transition can be represented by the membership function $\mu(x)$, as shown in Fig. 1.

$$\mu(x) = \begin{cases} \mu_1(x) & x_i \le X_0 \\ \mu_2(x) & x_i > X_0 \end{cases} \tag{4}$$

where $\mu_1(x) \in [0,1]$ and $\mu_2(x) \in [0,1]$. The function $\mu(x)$ indicates that what degree the measured value x_i conforms to the set A.

The true value X_0 is unknown, and the mathematical expectation (in statistical theory) or the fuzzy expectation (in fuzzy mathematics) can be used to estimate X_0.

In Fig. 1, x_v is the value of x when $\mu(x)$=1, which can be used to estimate the value of X_0 as follows:

$$X_0 \approx x\big|_{\mu(x)=1} = x_v \tag{5}$$

From Fig. 1, $\mu_1(x)$ is an increasing function, whereas $\mu_2(x)$ is a decreasing function. If $\lambda \in [0,1]$ as the level λ then $\mu_{A\lambda} = \lambda$, and the interval of x associated with the set A is obtained as follows:

$$U_{F\lambda} = x_U - x_L = s_1 + s_2 \tag{6}$$

In (6), $U_{F\lambda}$ stands for the interval of x associated with the set A, x_L and x_U can be determined from (7) and (8), respectively, and s_1 and s_2 are two intervals on either side of X_0 along the x axis that they can define the width of the membership function at the level λ, which can be represented as

$$\min\left|\mu_1(x) - \lambda\right|_{x=x_L} \tag{7}$$

$$\min\left|\mu_2(x) - \lambda\right|_{x=x_U} \tag{8}$$

For the measured value x_i, if $\lambda = \lambda^*$ is given, then $U_{F\lambda} = U_{F\lambda^*}$ is uniquely determined, i.e., the range of the scatter of the measured values x_i about the true value X_0 is given by $U_{F\lambda^*}$.

In Fig. 1, B is the fuzzy interval, λ^* is the optimal level, and $U_{F\lambda^*}$ is the fuzzy practicable interval at the level λ^*. Thus, the characteristic function $G_{A\lambda}(x)$ can be defined as

$$G_{A\lambda}(x) = \begin{cases} 1(\text{true}) & \mu_A(x) \ge \lambda^* \\ 0(\text{false}) & \mu_A(x) < \lambda^* \end{cases} \tag{9}$$

Equation (9) shows that usable values of x, which lie in the interval $U_{F\lambda^*}$, are represented by 1 (true), whereas those values that outside of this interval are unusable and are represented by 0 (false).

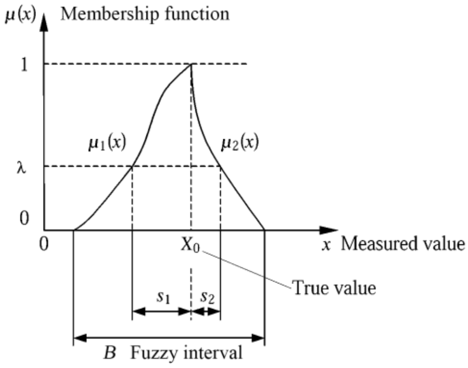

Fig. 1. The membership function and measured values

According to the theory of measurement uncertainty, the expanded uncertainty of measured results can be characterized using the fuzzy practicable interval $U_{F\lambda^*}$.

B. The theoretical value of λ^*

With the aid of fuzzy mathematics, λ^* can be used to confirm the limits of the range of an entity, between one extreme and another. In reality, λ^* can be regarded as a fuzzy number, and its fuzzy nature is at a maximum when λ^*=0.5, representing both true and false. $\lambda \geq 0.5$ means that the set A contains the most usable x values. Therefore, in theory, λ^* can be assumed be 0.5. In practical data analysis, generally, λ^* =0.4~0.5. λ^* =0.4 tends to correspond to the case in which n is fairly small.

C. Parameter mapping

The membership function in fuzzy mathematics can be represented by a probability density function in error theory. If the probability density function $p=p(x)$ is known, then the linear transformation

$$\mu(x)=\left(p(x)-p_{\min}\right)/\left(p_{\max}-p_{\min}\right) \quad p_{\max}\neq p_{\min} \tag{10}$$

can be used to map p onto the interval [0,1]; thus, $\mu(x)$ can be obtained. According to (5) and (10), x_v corresponds to $p_{\max}$, which is the maximum value of the probability density function.

Because x_i is a fuzzy number, its value lies on the interval [0,1]. Therefore, the linear transformations

$$\eta_v=\left(x_v-x_{\min}\right)/\left(x_{\max}-x_{\min}\right) \tag{11}$$

$$\eta(x)=\left(x-x_{\min}\right)/\left(x_{\max}-x_{\min}\right) \tag{12}$$

$$\tau=\tau(x)=\left|\eta(x)-\eta_v\right|=\left|x-x_v\right|/\left|x_{\max}-x_{\min}\right| \tag{13}$$

can be used to map x_i onto the interval [0,1]; then, the measured value is represented by the fuzzy number $\tau(x)$, where x_v corresponds to τ_v=0.

On the interval [0,1], $U_{F\lambda}$ is expressed by $\Phi_{F\lambda}$, s_1 and s_2 are expressed by ξ_1 and ξ_2, respectively. Equation (6) can be expressed as

$$\begin{aligned} U_{F\lambda} &= s_1+s_2 \\ &=\left|x-x_v\right|_{\mu_1(x)=\lambda}+\left|x-x_v\right|_{\mu_2(x)=\lambda} \\ &=\left(x_{\max}-x_{\min}\right)\left(\tau|_{\mu_1(\tau)=\lambda}\right)+\left(x_{\max}-x_{\min}\right)\left(\tau|_{\mu_2(\tau)=\lambda}\right) \\ &=\left(\tau|_{\mu_1(\tau)=\lambda}+\tau|_{\mu_2(\tau)=\lambda}\right)\left(x_{\max}-x_{\min}\right) \\ &=\left(\xi_1+\xi_2\right)\left(x_{\max}-x_{\min}\right)=\Phi_{F\lambda}\left(x_{\max}-x_{\min}\right) \end{aligned} \tag{14}$$

with

$$\Phi_{F\lambda}=\xi_1+\xi_2 \tag{15}$$

Hence, Fig. 1 can be further enlarged to become Fig. 2.

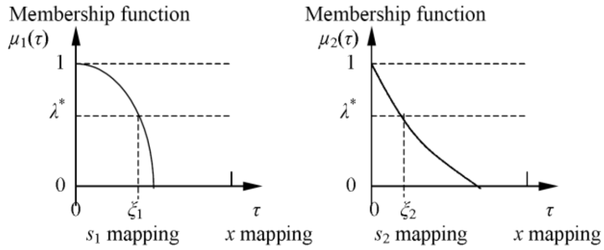

Fig. 2. Membership functions and mapping

If the discrete values $\mu_{1j}(\tau_j)$ and $\mu_{2j}(\tau_j)$, j=1,2,..., are known, then $\mu_1(\tau)$ and $\mu_2(\tau)$ can be obtained using the following maximum-norm method.

Define the ∞-norm as follows:

$$\|r\|_\infty=\max\left|r_j\right| \qquad j=1,2\ldots \tag{16}$$

Next, construct the polynomials $f_1(\tau)$ and $f_2(\tau)$

$$f_1(\tau)=\mu_1(\tau)=1+\sum_{l=1}^{L}a_l\tau^l \tag{17}$$

$$f_2(\tau)=\mu_2(\tau)=1+\sum_{l=1}^{L}b_l\tau^l \tag{18}$$

where $f_1(\tau)$ and $f_2(\tau)$ approximate the discrete values $\mu_{1j}(\tau_j)$ and $\mu_{2j}(\tau_j)$, respectively.

If

$$r_{1j}=f_1\left(\tau_j\right)-\mu_{1j}\left(\tau_j\right) \qquad j=1,2,\ldots v \tag{19}$$

$$r_{2j}=f_2\left(\tau_j\right)-\mu_{2j}\left(\tau_j\right) \qquad j=v,v+1,\ldots n \tag{20}$$

then select $a_l=a_l^*$ that satisfies

$$\min\|r_1\|_\infty \tag{21}$$

and select $b_l=b_l^*$ that satisfies

$$\min\|r_2\|_\infty \tag{22}$$

Thus, the unknown coefficients a_l and b_l can be computed.

In (17) and (18), quite a high approximation accuracy can generally be achieved when the degree L of polynomials $f_1(\tau)$ and $f_2(\tau)$ is equal to 3 or 4.

The constraint conditions of (21) and (22) can be given as follows:

$$f_1^{'}(\tau)=\mathrm{d}f_1(\tau)/\mathrm{d}\tau\leq 0 \tag{23}$$

$$f_1^{'}(\tau)=\mathrm{d}f_1(\tau)/\mathrm{d}\tau\leq 0 \tag{24}$$

which reveal the monotonic nature of the membership functions.

The approximation accuracy of the maximum-norm method is higher than that of the least-squares method. In addition, ξ_1 and ξ_2 can be solved using the following two equations:

$$\min\left|\mu_1(\tau)-\lambda^*\right|_{\tau=\xi_1} \tag{25}$$

$$\min\left|\mu_2(\tau)-\lambda^*\right|_{\tau=\xi_2} \tag{26}$$

The confidence level P is written as

$$P=\frac{\int_0^{\xi_1}f_1(\tau)\mathrm{d}\tau|_\lambda+\int_0^{\xi_2}f_2(\tau)\mathrm{d}\tau|_\lambda}{\int_0^{\xi_1}f_1(\tau)\mathrm{d}\tau|_{\lambda=0}+\int_0^{\xi_2}f_2(\tau)\mathrm{d}\tau|_{\lambda=0}}\times 100\% \tag{27}$$

In (27), $|_\lambda$ indicates that the operations are performed at the level λ. Equation (27) must satisfy $0 \le P \le 1$.

According to (27), the confidence level P is influenced by the level λ and the degree L of polynomials $f_1(\tau)$ and $f_2(\tau)$. In practical computations, the confidence level P is generally given. By giving preference to L=3, referring to the discussion of the theoretical value of the parameter λ^* given in section 2.2, and adjusting λ to satisfy P, the optimal level λ^* and the fuzzy practicable interval $U_{F\lambda^*}$ can be determined for a given confidence level P.

D. Establishment of linear membership functions

There are two common types of methods for establishing membership functions, i.e., the right-square-graph estimation method and the sort linear estimation method. The latter is applied in this model.

Sort the time series X_r in ascending order to construct a new data sequence X:

$$X = \{x_1, \ldots, x_i, \ldots, x_n\}; x_i \le x_{i+1} \tag{28}$$

The difference in value between adjacent data points is defined as

$$\Delta_i = x_{i+1} - x_i \ge 0 \tag{29}$$

The smaller Δ_i is, the thicker is the distribution of the measured values. Conversely, as Δ_i becomes larger, the distribution becomes thinner. This behaviour illustrates that Δ_i is related to the distribution density of x_i.

In view of the difference value sequence Δ_i, the linear membership function is defined as

$$m_j = 1 - (\Delta_j - \Delta_{\min}) / \Delta_{\max} \tag{30}$$

which is the approximate probability distribution density factor with

$$\Delta_{\max} = \max_{j=1}^{n-1} \Delta_j \tag{31}$$

$$\Delta_{\min} = \min_{j=1}^{n-1} \Delta_j \tag{32}$$

According to fuzzy-set theory, let the maximum probability distribution density factor be $m_{\max}$, let the x_i corresponding to $m_{\max}$ be x_v, and let the number i be v. If there are t repeated instances of $m_{\max}$, then x_v and v can be determined by the arithmetic mean algorithm. Thus,

$$p_{1j}(x_j) = m_j \quad j = 1, 2, \ldots, v \tag{33}$$

$$p_{2j}(x_j) = m_j \quad j = v, v+1, \ldots, n \tag{34}$$

and $\mu_{1j}(\tau_j)$ and $\mu_{2j}(\tau_j)$ can then be obtained using (10)-(24).

E. The steps for computing the expanded uncertainty

(i) Begin with the measurement sample $X_r=\{x_r(k)\}$; k=1, 2,..., n; r=1, 2,..., R;

(ii) Sort the time series X_r in ascending order to construct the new data sequence $X=\{x_1,..., x_i,..., x_n\}$; $x_i \le x_{i+1}$;

(iii) After obtaining v and x_v, compute $p_{1j}(x_j)=m_j$ (j=1, 2,..., v) and $p_{2j}(x_j)=m_j$ (j=v,v+1,..., n) using (29)-(34);

(iv) After computing $\mu_{1j}(\tau_j)$ (j=1, 2,..., v) and $\mu_{2j}(\tau_j)$ (j=v, v+1,..., n) using (10), compute $\eta(x)$ and τ according to (12) and (13);

(v) Under the constraint conditions of (23) and (24), establish the mathematical models for $f_1(\tau)$ and $f_2(\tau)$in accordance with (17), (18) and (19)-(22);

(vi) Obtain the membership functions $\mu_1(x)$ and $\mu_2(x)$ according to (17) and (18);

(vii) Let the confidence level be P=90%, and let the degree of the polynomials be L=3; then, adjust λ, compute ξ_1 and ξ_2 at the level $\lambda=\lambda^*$ using (25) and (26), and obtain the fuzzy practicable interval $U_{F\lambda^*}$, which is the expanded uncertainty of the measurement values according to (14).

III. Case Studies

A. Simulation tests

The purpose of the simulation tests is to check the validity of the fuzzy norm method proposed in this paper.

Suppose that the rolling bearing vibration performance obeys some known probability distributions, such as normal distribution, uniform distribution, triangular distribution, and Rayleigh distribution. A large number of measured values are simulated using the computer, and the expanded uncertainty of the measured values is estimated by the statistical method and the fuzzy norm method, respectively. In order to verify the effectiveness of the fuzzy norm method, the result estimated using the statistical method is considered as the true value and the relative error dU can therefore be defined by

$$\mathrm{d}U = \frac{|U_{F\lambda^*} - U_{\mathrm{True}}|}{U_{\mathrm{True}}} \times 100\% \tag{35}$$

In (35), U_{True} is the extended uncertainty estimated using the statistical method and $U_{F\lambda^*}$ is the extended uncertainty estimated using the fuzzy norm method.

Normal distribution 1024 measured values which obey the normal distribution with the standard deviation σ=0.0979 were generated by the computer simulation, as shown in Fig. 3. In this figure, $x(k)$ is the kth measured value, and k is sequence number and k=1,2,…,1024. Let the confidence level P=99.73%=0.9973, then the expanded uncertainty of the measured values was estimated by the statistical method and the result was

$$U_{\mathrm{True}}=6\sigma=0.5874,$$

and the expanded uncertainty of the measured values was estimated using the fuzzy norm method and the result was $U_{F\lambda^*}$=0.50338. According to (35), the relative error between the results estimated by the two methods is dU =14.3%.

Uniform distribution 1024 measured values which obey the uniform distribution with the standard deviation σ=0.28312 were generated by the computer simulation, as shown in Fig. 4. In this figure, $x(k)$ is the kth measured value, and k is sequence number and k=1,2,…,1024. Let the confidence level P=100%=1, then the expanded uncertainty of the measured values was estimated by the statistical method and the result was

$$U_{\mathrm{True}}=2(P-0.5)\sqrt{12}\sigma=0.98076,$$

and the expanded uncertainty of the measured values was estimated using the fuzzy norm method and the result was $U_{F\lambda^*}$=0.95731. According to (35), the relative error between the results estimated by the two methods is dU =2.39%.

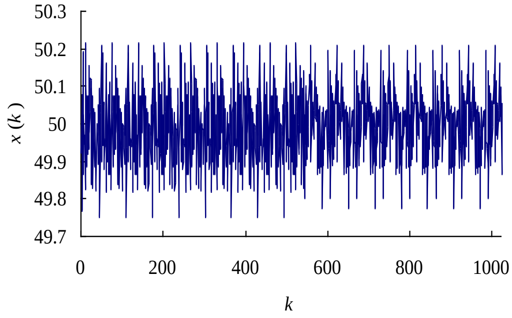

Fig. 3. Normal distribution data sequence

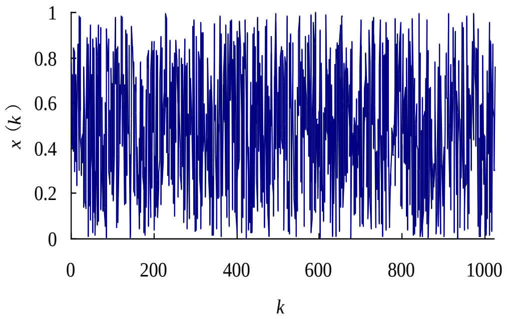

Fig. 4. Uniform distribution data sequence

Rayleigh distribution 1024 measured values which obey the Rayleigh distribution with the standard deviation σ=0.67974 were generated by the computer simulation, as shown in Fig. 5. In this figure, $x(k)$ is the kth measured value, and k is sequence number and k=1,2,…,1024. Let the confidence level P=99.73% =0.9973, then the expanded uncertainty of the measured values was estimated by the statistical method and the result was

$$U_{\mathrm{True}}=2\times2.636\sigma=3.58359,$$

and the expanded uncertainty of the measured values was estimated using the fuzzy norm method and the result was $U_{F\lambda*}$=3.92305. According to (35), the relative error between the results estimated by the two methods is dU =9.47%.

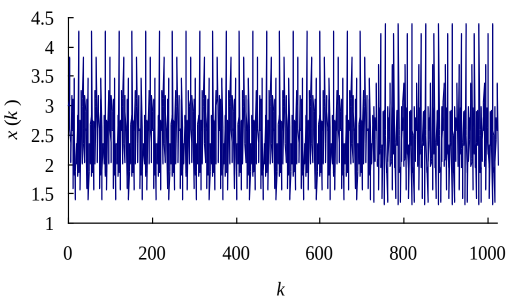

Fig. 5. Rayleigh distribution data sequence

Triangular distribution 512×2=1024 measured values which obey the double half-triangular distribution with the standard deviation σ=0.207047 were generated by the computer simulation, as shown in Fig. 6. In this figure, $x(k)$ is the kth measured value, and k is sequence number and k=1,2,…,1024. Let the confidence level P=100%=1, then the expanded uncertainty of the measured values was estimated by the statistical method and the result was

$$U_{\mathrm{True}}=2\left(\sqrt{6}-\sqrt{6(1-P)}\right)\sigma=1.01432,$$

and the expanded uncertainty of the measured values was estimated using the fuzzy norm method and the result was $U_{F\lambda*}$=1.02704. According to (35), the relative error between the results estimated by the two methods is dU =1.25%.

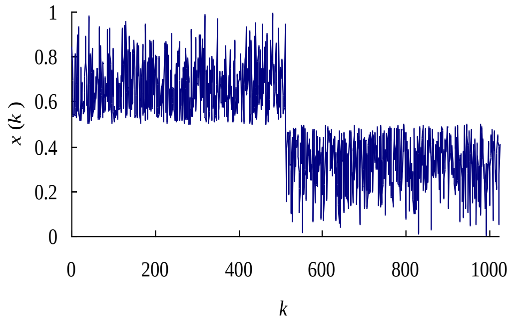

Fig. 6. Double half-triangular distribution data sequence

The simulation tests show that with regard to the known probability distributions, the relative errors between the results estimated by the statistical method and the fuzzy norm method are very small. The maximum of the relative errors is 14.3% and the minimum of the relative errors is 1.25%. This indicates that results estimated by the fuzzy norm method are in good agreement with results estimated by the statistical method, thus proving the validity of the fuzzy norm method proposed in this paper.

It is worth noting that, in the simulation tests above, it is easy to see that if the measured values are of different probability distributions, different equations are required for estimating the expanded uncertainty using the statistical method. It means that the results estimated by the statistical method depend on the known probability distribution, and if the probability distribution of the measured values is unknown, the statistical method becomes invalid. This is a gap in the statistical method. However, the fuzzy norm method proposed in this paper does not depend on any probability distribution and allows probability distribution unknown. Therefore, the fuzzy norm method proposed in this paper can fill this gap in the statistical method.

In fact, so far, the probability distribution of the rolling bearing vibration performance is still unknown. Thus, it is difficult to estimate the error produced by assessing the expanded uncertainty in rolling bearing vibration performance using the statistical method.

B. Practical case

An experiment to investigate rolling bearing performance was conducted on a custom-designed experimental platform. The experimental data (m/s^2) for the vibration acceleration of the rolling bearing were measured using an acceleration sensor, whose measurement principle is illustrated in Fig. 7.The experiment was performed from November 8, 2010 to December 23, 2010. The operating conditions were as follows: the axial loading was 19.6 N, and the speed of the inner ring of the bearing was 1000 r/min. The bearing data were collected regularly every day.

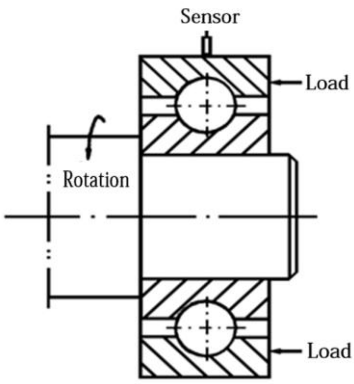

Fig. 7. Measurement principle of the acceleration sensor

The experimental data were divided based on the day on which they were collected. The data from November 8th, 13th, 18th, 23rd, and 28th and December 3rd, 8th, 13th, 18th, and 23rd were selected. Therefore, the final data selected for analysis were collected every 5 days on a total of 10 different days.

Moreover, the first 1000 data points collected on each of these days were used as the 10000 original vibration data points for the entire bearing experiment, i.e., the experimental data corresponded to a total of R=10 time units and each time unit corresponded to n=1000 data. The time series of the vibration information were thus obtained as shown in Fig. 8.

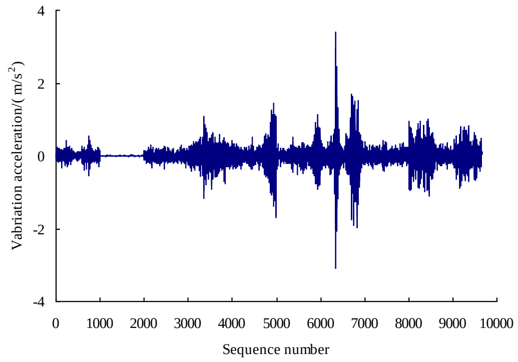

Fig. 8. Time series X_1-X_{10} of vibration information

As mentioned above, the confidence level P depends on the level λ and the degree L of the polynomials $f_1(\tau)$ and $f_2(\tau)$. In this case, P=90% and L=3 were chosen; thus, λ was adjusted to obtain λ^* such that P=90% was satisfied. The results obtained for the optimal level λ^* are shown in Table 1.

Table 1. The optimal levels λ^* corresponding to P=90% and L=3 in the 10 investigated time intervals.

r	1	2	3	4	5	6	7	8	9	10
λ^*	0.5895	0.459	0.557	0.6	0.523	0.622	0.7135	0.521	0.728	0.55

Following the steps for the computation of the expanded uncertainty presented in section 2.5, ξ_1 and ξ_2 were obtained for the levels $\lambda=\lambda^*$ as specified in the table; then, the measured true value X_0 was also obtained, and the values of the daily upper and lower bounds X_U and X_L on the vibration acceleration of the rolling bearing were computed according to (13). The results of these computations are shown in Fig. 9.

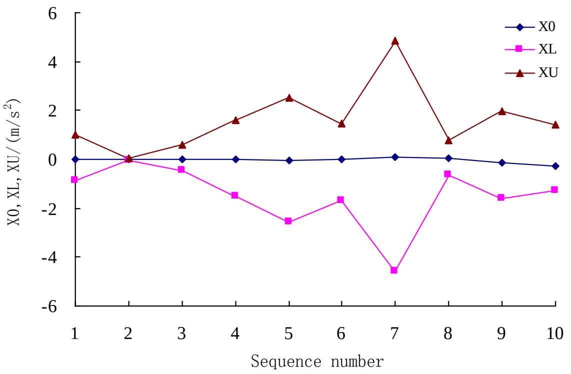

Fig. 9. The true values and upper and lower bounds of the 10 vibration-acceleration time series

The expanded uncertainty of the vibration performance of the rolling bearing at the optimal level λ^* was then obtained, as shown in Fig. 10.

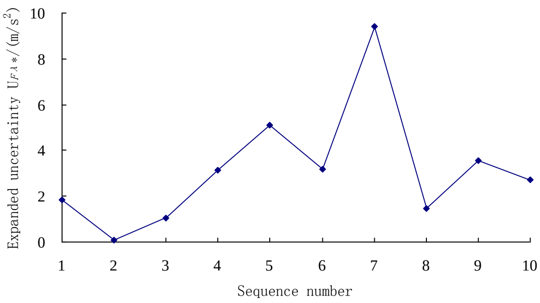

Fig. 10. The expanded uncertainties of the 10 vibration-acceleration time series

By comparing Fig. 9 and Fig. 10, the following conclusions can be drawn:

In Fig. 9, the estimated true values of the vibration acceleration of the rolling bearing remain essentially unchanged for all time series.

For r =1~3 (i.e. from the first day to the third day), the rolling bearing vibration process appears rather stable relative to the corresponding estimated true values. Additionally, the fluctuation interval and the uncertainty in the vibration acceleration are also both quite stable; the uncertainties for these three days are 1.8531, 0.0776 and 1.0257, respectively, indicating no remarkable variation in vibration performance.

For r =4~7 (i.e., from the 4th day to the 7th day), the rolling bearing vibration process exhibits gradually increasing fluctuations relative to the corresponding estimated true values, i.e., the fluctuation interval and the uncertainty both increase over time, reaching the maximum on the 7th day. The uncertainties for these 4 days are 3.1213, 5.0954, 3.1609 and 3.1213, respectively, exhibiting a dramatic increase and indicating a remarkable variation in vibration performance compared with that of the first three days.

For r =8~10, (i.e. from the 8th day to the 10th day), the rolling bearing vibration process again appears fairly stable relative to the corresponding estimated true values, with uncertainties of 1.4493, 3.5387 and 2.7066 for days 8, 9 and 10, respectively. These results again indicate an insignificant variation in vibration performance.

In Fig. 9, the estimated interval [X_L, X_U] represents the fluctuation interval of the vibration-acceleration time series relative to the true value X_0. In Fig. 10, the expanded uncertainty $U_{F\lambda*}$ represents the uncertainty of the vibration-acceleration time series. From Fig. 9 and Fig. 10, it can be observed that the variation in rolling bearing vibration performance is a dynamic, nonlinear, complex and unknown process.

Most importantly, in this experiment, the probability distribution and variational trends in the vibration acceleration of the rolling bearing were unknown. No a priori knowledge was available except the vibration-acceleration time series. Even when such knowledge is incomplete, the fuzzy norm method can be used to precisely evaluate the uncertainty in the vibration performance of a rolling bearing to accurately evaluate the vibration conditions of the bearing.

IV. Conclusions

Evaluation of uncertainty in rolling bearing vibration performance in this paper can be realized by the fuzzy norm method, without considering the probability distribution of rolling bearing vibration performance unknown.

In this paper, four kinds of simulation tests show that under the condition of unknown probability distribution, it fully verified the effectiveness of the fuzzy norm method. In the practical case, the fuzzy norm method was adopted to evaluate the uncertainty of the vibration performance of a rolling bearing using a fuzzy practicable interval, i.e., the expanded uncertainty, in the case that the probability distribution and variational trends of the vibration acceleration of the rolling bearing are unknown. The magnitude of the expanded uncertainty can reveal the degree of variation in the vibration performance of a rolling bearing to allow for accurate evaluation of the vibration conditions of the bearing.

In conclusion, the fuzzy norm method was applied to evaluate the uncertainty in rolling bearing vibration performance and to evaluate the uncertainty in the measured values of other system properties in this paper.

Conflict of Interest

The authors clearly acknowledge that this paper has no any potential conflict of interest.

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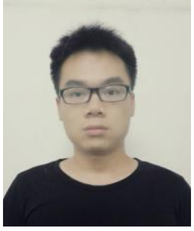

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